Re: Convergence of random variables

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- *Date*: 19 Apr 2005 19:52:22 −0500

In article <14341314.1113939049192.JavaMail.jakarta@xxxxxxxxxxxxxxxxxxxxxx>,
quantalfred <quantalfred@xxxxxxxxx> wrote:
>When talking about almost sure convergence, do we have to clarify the underlying sample space beforehand?

>Say for example, let X_n be iid such that
>P(X_n)=1=P(X_n)=0=1/2,
>then what's lim X_n?

>It looks to me that the answer is X_1. However, by the Borel−Cantelli lemma (second), we conclude that X_n=1 io and X_n=0 io. And lim X_n is a tail function, so P(limsup X_n = 1) is either 0 or 1. But all these two things are not true if we clarify the probability space, say ([0,1], B[0,1], Lebesgue measure), then if we set X_n(x) = 1 if 0<=x<1/2, X_n(x) = 0 if 1/2<=x<=1, we have P(x: limsup X_n(x) = 1) = 1/2.

>What's wrong there? I'm really confused.

For the limit of the X_n to exist, as the space is
discrete, all the X's after some point have to be equal.
Given the probability distributions and independence, this
has probability 0, so the limit exists almost nowhere.
The limit also cannot exist in probability, as this would
mean, in this discrete case, that two X's for large
indices would have to be equal with large probability,
and they are only equal with probability 1/2.

The distributions being identical, the limit of the
distributions is this common distribution, and this
is called convergence in distribution. Convergence
in distribution does not imply convergence in
probability unless the limiting distribution puts all
the probability at one point.
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This address is for information only. I do not claim that these views
are those of the Statistics Department or of Purdue University.
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- ***References***:
    - ***Convergence of random variables***
        - *From:* quantalfred

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