Re: polynomial algebra question

Source: http://sci.tech−archive.net/Archive/sci.math/2005−04/msg03346.html

- *From*: "Chip Eastham" <hardmath@xxxxxxxxx>
- *Date*: 22 Apr 2005 08:07:32 −0700

John H Palmieri wrote:
> On Apr 19 2005, "Chip Eastham" <hardmath@xxxxxxxxx> wrote:
>
> > John H Palmieri wrote:
> >> Suppose K/F is a field extension, A is a commutative F−algebra,
and
> >> (K tensor_F A) is isomorphic to K[x]. Must A be isomorphic to
F[x]?
> >>
> >> I know that the answer is yes with some added hypotheses, but I'm
> >> wondering if it's true without them.
> >
> > Hi, John:
> >
> > Do we not need to specify the sense in which (K tensor_F A) is
> > isomorphic to K[x}?
> >
> > Surely an isomorphism as F−modules (vector spaces) is too little,
as
> > this can only guarantee equality of dimensions.
> >
> > regards, chip
>
> Isomorphic as K−algebras.

Thanks, John. One can show this property of field extensions is
transitive.

That is, define a field extension K/F to be Palmieri−flat (or P−flat)
iff:

For any F−algebra A, K tensor_F A is isomorphic as a K−algebra to K[X}
implies A is isomorphic as an F−algebra to F[X].

Then K/H is P−flat and H/F is P−flat imply that K/F is P−flat.

Proof: A calculation based on K tensor_H (H tensor_F A) being
isomorphic as a K−algebra to K tensor_F A gives H tensor_F A isomorphic

to H[X] by P−flatness of K/H, and hence that A isomorphic to F[X] by P−flatness of H/F.

If A is algebraic over F, then K tensor_F A is algebraic over K. So a field extension K/F which is not P−flat must involve a counterexample A that is not algebraic over F.

No simple extension A = F[a] can be a counterexample (a must either be transcendental or algebraic wrt F), and from the above argument we cannot have a counterexample from finite extensions either.

regards, chip

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- ***References***:
  - ***polynomial algebra question***
    - ◊ *From:* John H Palmieri
  - ***Re: polynomial algebra question***
    - ◊ *From:* Chip Eastham
  - ***Re: polynomial algebra question***
    - ◊ *From:* John H Palmieri

- Prev by Date: ***Re: this is probably an easy question but...***
- Next by Date: ***Re: " solving h(3*x^ 2 + 2*x + 1) = { h(x)} ^ 2 + k ; h and k unknown "***
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