Re: Where do I begin?

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- *Date*: Sat, 30 Apr 2005 08:18:57 EDT

> > Consider the related series
> >
> > sum ((−1)^n)/(ln n + cos (2 \pi n p/q))
> >
> > where p/q is a fraction in lowest terms.
> > If q is odd this converges.
> > If q is even it seems to diverge;
> > known to be true for q=2, 4, 6, 8, but I haven't
> > proven
> > it for general even q.
> >
> > Don Coppersmith
>
> Don, thank you so much, I think you are right that it
> has something to do with this.
> Can you explain how you know that it converges for
> q=2, 4, 6, 8 etc..

Let q be even; we will show that it DIverges.
Pick some large m, a multiple of q.
Consider the terms for n=m+1,m+2,...,m+q.
log n differs from log m by O(1/m),
which we will be able to ignore.
For shorthand set x=log m.
In that range, consider
sum (−1)^n / (log n + cos (2 \pi n p/q)).
Incur an error O(1/(m log m)) in each term
by replacing log n by x.
The sum of these q terms is then
sum[k=1..q] (−1)^k /(x+cos(2 \pi k p/q)) +O(q^2/m log m).
Without the error term,
that sum is a nonzero fraction in x;
its absolute value is at least
1/x^{1 + q/2} = 1/(log m)^{1 + q/2}.
This exceeds the error term.
It provides consistent bias in one direction;
for every q terms of the original series, we get a
bias of about 1/(log m)^(1+q/2).
The sum of those biases diverges.

Do you also see why it CONverges for odd q?

Don Coppersmith
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