

Re: abundance of irrationals!)

Source: <http://sci.tech--archive.net/Archive/sci.math/2005-05/msg00944.html>

- *From:* Tony Orlow (aeo6) <aeo6@xxxxxxxxxxxx>
 - *Date:* Thu, 5 May 2005 12:44:21 -0400
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Dave Rusin said:

>
> Tony has more gripes than I can deal with today but he asked for the
> mathematical definition of two terms and since I have students' topics
> on the brain this week I will oblige him on both.
>
>
> In article <MPG.1ce2ec19ab7de659989bb6@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> Tony Orlow (aeo6) <aeo6@xxxxxxxxxxxx> wrote:
>> Dave Rusin said:
>
>>> mathematicians might prefer to treat "infinite X" as a
>>> single noun phrase to be defined
>
>> Is there an agreed on definition of "infinite" that enables us to know what
>> each other is talking about?
>
> We say a set A is infinite if there is an injection $A \rightarrow A$ which is not onto.
>
> (That defines "infinite set". It doesn't define "infinite elephant"; I don't
> know what that would mean. It doesn't define "infinite integer"; I don't know
> what that means either, though I can usually try to get the gist of what
> people mean when they use it -- they mean something which is not, in fact,
> an integer, but that sort of thing may be permitted among friends! After all,
> a "red herring" is not red nor a herring -- it's a figure of speech.)
Well, this is exactly what I am asking. When we talk about finite vs. infinite
numbers, how can we even discuss this if we don't know what the word means?
Really, the set-based approach to infinity is not the only one, and it is
inappropriate for set theorists to ignore the nature of the elements they are
enumerating, when drawing conclusions about those elements or the set thereof.
This is why I think bijections are not the whole story, even if they are useful
in distinguishing some things.
>
> Some people do it differently: they say the previous definition should
> be of the phrase "Dedekind infinite", and then they define a companion
> phrase like, say, "Cardinal infinite" as follows:
> Definition 1. A set A is Cardinal-finite if there is a bijection
> $A \rightarrow \{ 1, 2, 3, \dots, n \}$ for some natural number n .
> Definition 2. A set A is Cardinal-infinite if it is not Cardinal-finite.

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>

> Then you get to do some interesting mathematics:

> Theorem 1. If a set A is Dedekind-infinite then it is Cardinal-infinite.

> Theorem 2. If a set A is Cardinal-infinite then it is Dedekind-infinite

> if the Axiom of Choice is accepted.

>

> (That is, the proof of Theorem 2 uses the Axiom of Choice. Is the Axiom of

> Choice "true"? The answer to that will be the same as the answer to the

> question, 'Is Theorem 2 "true"?')

>

> Most people are happy to accept the axiom of choice, which means we have

> a pair of theorems here which states that a set is Cardinal-infinite iff it

> is Dedekind-infinite, and sets which have either (hence both) property are

> then just called "infinite". Probably if you're in the camp that does not

> use the Axiom of Choice then you use "Cardinal-infinite" as a synonym

> of "infinite" and merely discuss "Dedekind-infinite" sets as curiosities.

> I wouldn't really know, not being in that camp.

I don't tend to deal with these axioms much. The only reason I am dealing with them now is that mathematicians seem not to be able to do without them. I have come to realize while in these newsgroups that the system which has always bothered me has discernible flaws which seem to go unnoticed, and that the incorrect conclusions of this theory seem to reverberate in people's minds, causing what I think is an incomplete and skewed understanding of infinity.

Hopefully this weekend I can get some time to continue fixing the axiomatic system and/or developing spectral infinities.

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>

>>> For example, the (correct) statement

>>> the limit of $f(x) = x \sin(1/x)$ as x tends to 0 is 0.

>>> may be expressed as a cumbersome but precise statement about the

>>> behaviour of the function f near zero. The words "end" or "stop"

>>> never occur in the translated statement.

>

>>near zero, or at zero? Most limits give pretty precise values for what is

>>happening at that precise point.

>

> Oh? What is "happening at [the] precise point" $x=0$ for this function f ?

> Are you saying the limit at $x=0$ does not exist because $f(0)$ is undefined?

Well $f(0)$ is undefined, why? Because $\sin(1/0)$, or $\sin(\infty)$, is undefined. But,

we know that \sin is always between 1 and -1 and so $x \sin(1/x)$ at $x=0$ is

equivalent to $0 \cdot$ (some number between 1 and -1), which is always 0. We didn't

even really need limits for this one, but we can use limits in many places

where we don't have to, and the value of the limit at that point is also 0.

Coincidence? No, this is what limits do. In other more complicated situations

we need limits. If they approach a certain value, then we say that is the value

of the function at that point. Or, if we don't, then we might as well.

>

> To me there is no confusion: f is not defined at $x=0$. On the other

> hand the limit of f , as x tends to zero, is indeed $L = 0$ because

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> for every $\epsilon > 0$, $|f(x)| < \epsilon$, which means that the values of f
> can be kept smaller than any pre-assigned epsilon simply by keeping
> x close to (but of course different from) epsilon.

>
> See? I told you it was cumbersome -- and that's the informal version.

> The most formal version is:

> $\forall \epsilon > 0 \exists \delta > 0 ((|x| < \delta \text{ and } x \neq 0) \Rightarrow (|f(x)| < \epsilon))$

> which would be read as

> For every positive epsilon there is a positive delta such that for all
> nonzero x whose distance from 0 is less than δ we have that $f(x)$
> differs from 0 by less than epsilon.

>
> No "end". No "stop". No "happening at $x=0$ ". And while I'm at it let
> me note that I chose this f as my example because there is no
> "infinite" either, and because there's no "...but never equal to"
> either, in the sense that I most definitely DO have $f(x)$ equalling
> the limiting value $L = 0$ over and over again (namely for each of
> $x = 1/\pi, 1/(2\pi), 1/(3\pi), \dots$ we have $f(x) = 0$). More details,
> including definitions, motivations, examples, and theorems, can be
> found in any decent calculus book.

>
> This may or may not correspond to your intuitive notion of "limit".

> If it does, fine. If not, then please don't use the word "limit"
> when referring to the behaviours of functions as the inputs approach
> a specific point a (or as the inputs increase without bound) because
> that would cause confusion. Define YOUR notion precisely and then
> call it something else -- you can refer to the "Orlow-limit of f at a "
> or the "tendency of f near a " or as " $L^*(f, a)$ " or whatever.

> Make sure your definition is clear enough so that one can deduce
> from the definition what (e.g.) $L^*(x \sin(1/x), 0)$ is supposed to equal.

>
>
> You don't need to agree that mathematicians' language is appealing.
> You should know, though, that it is incredibly universal. On the
> rare occasions when someone will offer definitions notably different
> from mine, s/he will be able to prove that the definitions are
> logically equivalent to mine. This is sci.math here so I think it
> is quite reasonable to insist that common math terms not be used to
> refer to non-standard concepts; pick new names for those new concepts.
> (E.g. "natural numbers"="set satisfying Peano axioms", "real numbers"=
> ordered field with LUB property, etc.) If you're not sure what the
> math terms are, read a book (<-- best choice!) or ask here and some
> helpful person will answer a polite query.

I am still not really sure what your complaint about my use of the word
"limit" is. Mathematicians by convention will only speak of "approaching
infinity" or "tending toward infinity" and not "when $n=\infty$ ". I guess that's it.
I'm a little too intimate with infinity, and not reverent enough, I guess.

There seems to be an aversion to the idea of infinite numbers, because normal
math doesn't work on them. So what? So we discover the math that does.

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> dave

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>

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Smiles,

Tony

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- *Follow-Ups:*

- ◆ *Re: abundance of irrationals!*
◇ *From: Dik T. Winter*

- *References:*

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