Re: Cantor and the binary tree

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In article <1116939502.814879.192170@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
mueckenh@xxxxxxxxxxxxxxxx wrote:

> Cantor and the binary tree.
>
> If we accept that, in binary digits, SUM{n = 1 ... oo} 2^−n = 0.111...
> = 1
>
> then all the real numbers of the interval [0,1] are realized as
> infinite paths of the binary tree
> .
> 0 1
> 0 1 0 1
> .....
>
> too (read from top to bottom). Each number is given by a path
> stretching over infinitely many nodes (bits). All nodes (bits) of the
> tree are countable. The paths are not, according to Cantor's famous
> diagonal proof.

So far so good.
>
> But we find that, up to line number n, there are −1 + 2^(n+1) nodes
> whereas 2^n different paths arrive at and 2^(n+1) different paths
> spring off from line number n. Hence, in the enumerated domain, there
> is at most one more path than nodes. After leaving any finite line
> number n (if it is reasonable to make such a distinction) we can no
> longer apply these formulae. But we know that any new branching
> increases the number of paths by 1 and, by definition, the number of
> nodes by 1 too (because any branching is a node). Therefore, the number
> of paths always equals that of the nodes + 1. It is simply impossible
> to assume that one of these numbers becomes uncountably infinite while
> the other remains countably infinite.

One does not assume it, one proves it, as I have done several times.

Here is an outline of that proof:

Every node is represented by a terminating binary (starting at "." and

terminating at the node itself in the tree above) which is like a subset of the rationals, which are countable.

Every unending path is represented by a non−terminating binary (also starting at "." but never ending), which surject onto the real interval [0,1], and are thus as uncountable as the reals.

That WM choses to reject proofs that show him wrong does not invalidate such proofs.
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- ***Follow−Ups***:
  - ♦ ***Re: Cantor and the binary tree***
    - ◊ *From:* mueckenh

- ***References***:
  - ♦ ***Cantor and the binary tree***
    - ◊ *From:* mueckenh

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