

Re: Cantor and the binary tree

Source: <http://sci.tech--archive.net/Archive/sci.math/2005-05/msg04436.html>

- *From:* mueckenh@xxxxxxxxxxxxxxxxxxxx
 - *Date:* 24 May 2005 11:14:39 -0700
-

Dik T. Winter wrote:

> In article <1116939502.814879.192170@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>
mueckenh@xxxxxxxxxxxxxxxxxxxx writes:
>> If we accept that, in binary digits, $\text{SUM}\{n = 1 \dots \infty\} 2^{-n} =$
0.111...
>> = 1
>
> You may note that that is **not** an infinite sum...
>
>> .
>> 0 1
>> 0 1 0 1
>>

Any path is an infinite sequence of bits which by multiplying with 2^{-n} and summing up establishes an infinite series representing a real number. Every combination of countably many bits is realized by definition.

> ...
>> But we find that, up to line number n , there are $-1 + 2^{(n+1)}$ nodes
>> whereas 2^n different paths arrive at and $2^{(n+1)}$ different paths
>> spring off from line number n . Hence, in the enumerated domain, there
>> is at most one more path than nodes. After leaving any finite line
>> number n (if it is reasonable to make such a distinction) we can
no
>> longer apply these formulae. But we know that any new branching
>> increases the number of paths by 1 and, by definition, the number of
>> nodes by 1 too (because any branching is a node). Therefore, the number
>> of paths always equals that of the nodes + 1. It is simply impossible
>> to assume that one of these numbers becomes uncountably infinite while
>> the other remains countably infinite.
>

Re: Cantor and the binary tree

> Also we find that up to line n , summing up to that node along the path
> gives a value for $k/(2^n)$ for some integer k .

It is forbidden to stop there, by definition. If you want to realize a terminating rational with n bits, then you must follow, from line $n+1$ on, the path with infinitely many zero-bits.

Therefore a number along
> a node is always equal to a rational with a denominator that is a power
> of two. It is simply impossible to assume that one of these numbers
> becomes $1/3$.

Why should $0.010101\dots$ not exist in that tree? Every path is infinite by definition as is $0.010101\dots$, by definition.

Regards, WM

.

-
- *Follow-Ups:*
 - ◆ *Re: Cantor and the binary tree*
 - ◇ *From:* Dik T. Winter
 - ◆ *Re: Cantor and the binary tree*
 - ◇ *From:*