

Re: Cantor and the binary tree

Source: <http://sci.tech-archive.net/Archive/sci.math/2005-05/msg04894.html>

- *From:* Tony Orlow (aeo6) <aeo6@xxxxxxxxxxxx>
 - *Date:* Thu, 26 May 2005 09:57:52 -0400
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Virgil said:

> In article <1117020437.377622.305540@xx>,
> mueckenh@xxxxxxxxxxxxxxxxxxxx wrote:
>
>> Robert Kolker wrote:
>>> Tony Orlow (aeo6) wrote:
>>>>
>>>> That's all very well and good, if you specify f and g and figure those
>>>> functions into your comparison. It's a mistake to ignore them.
>>>
>>> Are you capable of following a proof? Even a three line proof?
>>
>> Are you capable to follow a five lines proof without referring to "Big
>> Brother" Cantor? Consistency of set theory is questioned, hence I do
>> not accept Cantor's proof as an argument.
>>
>> line number n
>> 0 0.
>> 1 0 1
>> 2 0 1 0 1
>>
>>
>>
>> 1) Each real number of (0,1) is given by a path stretching over
>> infinitely many nodes (bits).
>> 2) All nodes (bits) of the tree belong to a countable set.
>> 3) A node can only exist within a path.
>> 4) Any node increases the number of paths by 1 from 1 coming in, to 2
>> going out. $2 - 1 = 1$.
>> 5) Any node increases the number of nodes by 1.
>>
>> Please point out which step is wrong.
>
> WM's "proof" disproved"
>
> WM conflates bounded paths, having terminal or leaf nodes with unbounded
> unending paths which have no terminal or leaf nodes, but contain
> infinitely many intermediate nodes.
>

Re: Cantor and the binary tree

> 1) Each number of (0,1) is given by an UNENDING path stretching over
> infinitely many nodes (bits).

And yet you claim that no node is infinitely far from the root? Wrong.
besides, I thought you were objecting earlier to appending trailing
zeroes....maybe you changed your mind.

>

> 2) All nodes (bits) of the tree belong to a countable set.

Sure.

>

> 3) A node can only exist within a path.

Of course.

>

> 4) Any node increases the number of ENDING paths, having terminal or
> leaf nodes, by 1 from 1 coming in, to 2 going out. $2 - 1 = 1$.

You are really hung up on this lack of end. That's your problem, whether it has
to do with paths on a tree, or the "greatest finite natural". You need to look
where you can actually see something for once.

Anyway, one path is added for any TWO nodes. But do go on....

>

> 5) Any node increases the number of nodes by 1, but have absolutely
> nothing to do with the number of unending paths.

Actually, if you have an infinite binary tree, with all paths unending, it's
rather difficult to add a node, isn't it? I suggest we would have a more
fruitful conversation talking about inserting nodes in the middle of an
infinite tree, and see what that does for us.

>

>

> All unending paths in an unending binary tree contain infinitely many
> nodes.

Most of which are infinitely far from the root.

>

> The number of leaf nodes exactly equals the number of ending or finite
> paths in any finite binary tree (in which all paths end).

And which you have conveniently dispensed with for the sake of your argument.

Typical Cantorian prestadigitation.

>

> Considering the binary tree whose root is "." and each branch is
> indicated by a "0" or a "1", each leaf node, and therefore each path, is
> represented by a terminating binary fraction, but each unending path is
> represented by a non-terminating binary fraction.

>

> There are more of the non-terminating than of the terminating.

>

> So WM is wrong yet again.

>

And again you miss the point entirely, but will probably repeat the same
nonsense at least 6 more times, as is your tendency.

Let's put this baby to bed. See if you can poke a hole in this:

Re: Cantor and the binary tree

Let us consider the middle of a tree, finite or infinite, and what happens when we insert a node. We have the following tree, where each node has two children, $2x$ and $2x+1$ where the parent node is x :

```
1
2 3
4 5 6 7
8 9 10 11 12 13 14 15
```

Let's insert a node between 2 and 5. Sure, this will break the pattern I just mentioned; that was just so you could make sure you knew whose children were whose. We'll call our new node X , and say for argument that 5 will be its right child node. This leaves X with no left child node (I put a period where one would be):

```
1
2 3
4 X 6 7
8 9 . 5 12 13 14 15
10 11
```

Notice that we have not added a path yet, but only extended the paths that include the branch between 2 and 5, by one branch. Every time we add a node, we add a branch, but not necessarily a path. Not every node **MUST** have two children. In fact, if one creates a binary "tree" with no left nodes, we have a linked list which consists of a single path, even if it is infinitely long. It is only when we add a second child to a node that we create a new path. It takes two nodes to add a path. So, if we then add another element, Y , as the left child of X , **THEN** we have produced a new path, ending in Y , a new leaf node:

```
1
2 3
4 X 6 7
8 9 Y 5 12 13 14 15
10 11
```

So, it takes two nodes to add a path. Note that node 1 may have a parent node, and nodes 8,9,10,11,12,13,14,15 all may have children. It doesn't matter whether this is a finite tree, or an infinite one. It doesn't matter whether any leaf nodes existed previously, or whether the paths are finite or infinite. In all cases, the addition of a node adds one branch, and **MAY** add a path, but no more than one additional path for every two nodes or branches. Therefore, there are **ALWAYS** fewer paths than nodes, and the proposition that any binary tree has more branches than nodes is clearly wrong.

If someone wants to tell me that one can't insert a node in the tree, I suggest they ask their programmer friends, before wasting my time.

—
Smiles,

Tony
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• *Follow-Ups:*

- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Alan Morgan
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Virgil
- ◆ ***Re: Cantor and the binary tree***
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 ◇ *From:* Virgil
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Virgil
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Robert Kolker

• *References:*

- ◆ ***Cantor and the binary tree***
 ◇ *From:* mueckenh
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Robert Kolker
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Robin Chapman
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Ron Sperber
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* aeo6
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Robert Kolker
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* aeo6
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Robert Kolker
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* mueckenh
- ◆ ***Re: Cantor and the binary tree***
 ◇ *From:* Virgil

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