Re: Help in answering news story on refutation of fermat's last theorem

Source: http://sci.tech−archive.net/Archive/sci.math/2005−05/msg05433.html

- *From*: "Keith Ramsay" <kramsay@xxxxxxx>
- *Date*: 29 May 2005 14:03:15 −0700

anzaurres1@xxxxxxxxxxx wrote:
[...]
|4. There are no theories, studied by any professionally competent
|mathematician, which contain "false axioms", no mater how you
|define this term.
|
|Nor can there be one. Why? Because every theory, advanced in the
|science of mathematics, is based on at least one example, which
|satisfies all axioms of that theory.

I'm sorry, I don't mean this in a mean way, but you just don't know what you're talking about. You're basing your opinion on some inadequate experience of how "theories" in mathematics are studied, and then overconfidently generalizing your experience. Mathematical logicians are mathematicians, by the way. (And I also used to be a professional mathematician, in number theory.)

People have been known to study an axiom system before knowing whether there exists a model. There are some axiom systems motivated by quantum field theory, for example, for which the existence of a model is unknown.

Axiom systems are not always studied one by one, starting by being "advanced". Axiom systems are not always used as possible candidates for "working" systems, like ZFC or Quine's NF. There is plenty of work in mathematical logic that studies whole classes of axiom systems. Goedel's incompleteness theorems, for instance, aren't just about the system of Principia Mathematica, but about a whole class of models of which the system of Principia Mathematica is just a relatively famous example.

If an axiom system is consistent, people are not always interested in relating the axiom system to its own models; they may be more interested in some other model. When people study classes of axiom systems, they sometimes have a particular

language in mind, and stick to its original interpretation.
There are, then, notions of "truth" and "falsity" for the
statements of the axiom systems.

When Goedel published proofs of his incompleteness theorems,
he intentionally steered away from referring to such notions
as true or false statements of elementary arithmetic. It appears
he had concerns that reasoning with such concepts would cause
some of his audience to suspect some kind of subtle fallacy.
He wanted to make it clear that his reasoning was itself
combinatorial reasoning of an elementary kind, not problematic
in any way, so he dealt in more elementary, combinatorial
properties of his systems. Assuming that all the axioms of the
system are true is in a sense a more "complex" assumption.

Consider the omega−consistency assumption he made. An axiom system
is omega−inconsistent if there's a theorem of the form, "there
exists an n such that P(n)" deducible from the system, but for
each individual natural number n, that P(n) is false is a theorem.
Now, the only way that a theory can be omega−inconsistent is for it
to have a false theorem in it. Either one of the P(n) is actually
true, or else there doesn't actually exist an n such that P(n).

Why do you suppose he assumed that? Because he had not already
ruled out the possibility of some of the axioms being false.
Omega−consistency is a weaker requirement, but it's a more
"syntactic" one. Later on, Rosser took matters a step further
and generalized the result to axiom systems that are merely
consistent and not necessarily omega−consistent.

What additional cases are covered by Rosser's theorem than
Goedel's? Consistent but omega−inconsistent theories−− *all of
which have false theorems provable from them*. So Rosser's
work is an example of just the kind of thing you claim isn't
done. If we needed to, we could produce further examples.

|Thus, when you replied to me
|
|> > When we, mathematicians, say that a statement is true...

Why not slightly more context?

#When we, mathematicians, say that a statement is true in a given
#axiomatic system, we mean that one can logically derive this statement
#from the axioms.

Mathematicians talk about statements being true far, far
more often than they talk about statements being true in
a given axiom system.

|> People do indeed often speak... For example, it sometimes prompts

|> them to contradict the simple observation that there are
|> theories with false axioms

%People do indeed often speak of a statement being "true in a given
%axiomatic system" when they mean that it is provable in that system.
%While mostly harmless, this terminology promotes needless confusion.
%For example, it sometimes prompts them to contradict the simple
%observation that there are theories with false axioms.

|you were talking not about what professional mathematicians actually
do
|but you THINK professional mathematicians do. Your image of what
|professional mathematicians is dead wrong.

So what is it that you think is wrong about it?

You yourself referred to mathematicians talking about "true in an axiom system". If there's anything I would find doubtful about Torkel Franzen's remark here, it would simply be the word "often". I remember hardly ever hearing such usage from professional mathematicians. (I think that's because it's unfortunate usage.) I think I read a Paul Halmos essay where he used it that way. But it's entirely possible that the mathematicians you and Torkel Franzen have dealt with say this more often.

I'm not sure I'd concede that the usage is mostly harmless. But since we're talking about professionals, perhaps they are indeed able most of the time to avoid the kind of confusion that seems to plague the people who show up on usenet writing in terms of "truth in an axiom system".

I don't see any reasonable way around maintaining a distinction between truth and provability, even for a formalist. If I say that the Goldbach conjecture is true, I'm saying that a certain type of finite structure (an even number >2 but not expressible as a sum of two primes) does not exist. Goldbach's conjecture is equivalent to the impossibility of proving the existence of such a counterexample in a very simple axiom system. If I say that the Goldbach conjecture is provable from a given set of axioms, then I'm saying that a certain type of finite structure (a proof in the system) does exist. There's no way in general to make these mean the same thing.

Indeed, Goedel showed how to produce examples where a statement G is true if and only if it is not provable from an axiom system S, for a typical axiom system S. Certainly we could, if we wanted, continue to say "G is true in S" to mean G is provable in S, so that G being true would be equivalent to "G is not true in S". I just don't see any advantage in such a terminology.

Truth in a formal language is generally defined so that it's
equivalent to truth relative to a specific model. Truth in
elementary arithmetic for example is truth in the model N. Truth
in all models of a set of axioms has a good word for it too:
validity.

Keith Ramsay

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- ***Follow−Ups***:
  - ***Re: Help in answering news story on refutation of fermat's last theorem***
    - ◊ *From:* anzaurres1
  - ***Re: Help in answering news story on refutation of fermat's last theorem***
    - ◊ *From:* Torkel Franzen

- ***References***:
  - ***Re: Help in answering news story on refutation of fermat's last theorem***
    - ◊ *From:* anzaurres1
  - ***Re: Help in answering news story on refutation of fermat's last theorem***
    - ◊ *From:* Torkel Franzen
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    - ◊ *From:* anzaurres1
  - ***Re: Help in answering news story on refutation of fermat's last theorem***
    - ◊ *From:* Jesse F. Hughes
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    - ◊ *From:* Mitch Harris
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    - ◊ *From:* Russell

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  - *From:* anzaurres1

- Prev by Date: ***Re: Packing F−pentominoes into a 5x7x7 box***
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