

# Re: exponential equation with constant

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*Source:* <http://sci.tech-archive.net/Archive/sci.math/2005-06/msg00393.html>

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- *From:* [israel@xxxxxxxxxxx](mailto:israel@xxxxxxxxxxx) (Robert Israel)
  - *Date:* 2 Jun 2005 21:26:27 GMT
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In article <1117738290.754362.276360@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>, <quiasmox@xxxxxxxx> wrote:

> I had a problem in an electrical circuits class, which was to find the  
> point at which the voltage on two exponential curves is equal. These  
> were the decaying voltage on an inductor and the rising voltage on a  
> capacitor, the curves starting from a fixed voltage and from zero  
> (respectively) at the same instant. The equated formulas were  
>  $VL = 2 * \exp(-2000t) = 1 - \exp(-1000t) = VC$   
> By luck, I found I could solve it for t by turning it into a quadratic—  
> let  $y = \exp(-1000t)$   
>  $2y^2 + y - 1 = 0 = (2y-1)(y+1)$   
> Then, using  $(2y-1)$ ,  
>  $2 \exp(-1000t) = 1$ ,  
>  $\ln(2) = 1000t$   
>  $t = 693.2 \text{ us}$ .

> This solution depends on the coincidence that one exponent is exactly  
> twice the other.  
> This brings me to the question — is there a way, other than computer  
> simulation, to solve  $a * \exp(-ct) = 1 - b * \exp(-dt)$  for t?

Similar to your solution: let  $x = \exp(-ct)$ . Then you want to solve  
 $a * x + b * x^e = 1$  where  $e = d/c$ . I'll assume  $e > 1$  (if  $e < 1$  you can  
interchange the terms  $a * \exp(-ct)$  and  $b * \exp(-dt)$ ). In only a few cases  
you have a polynomial equation that can be solved using radicals.  
However, at least if  $b/(a^e)$  is small there is a series solution

$$x = \frac{1}{a} \sum_{n=0}^{\infty} (-1)^n \frac{\Gamma(n+1)}{(n! \Gamma(n(e-1)+2))} (b/a^e)^n$$

If e is rational, this can be expressed in terms of hypergeometric functions.

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- *Follow-Ups:*

- ◆ *Re: exponential equation with constant*  
◇ *From:* David W . Cantrell

- *References:*

- ◆ *exponential equation with constant*  
◇ *From:* quiasmox

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