Re: Cantor and the binary tree

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- *From*: Gottfried Helms <helms@xxxxxxxxxxxxx>

- *Date*: Sat, 04 Jun 2005 15:11:11 +0200

Well some thoughts, even if it is a bit late...

Am 29.05.05 17:40 schrieb mueckenh@xxxxxxxxxxxxxxxxx:

\>>
\>>No number can have the property of "being uncountable".
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\>
\> The number of elements of a set can be finite or infinte. It can be
\> countable (finite or aleph_0) or uncountable. In set theory such
\> numbers are defined, although you are correct.
\>
It seems to me, that this is a good point to pause and think about thinking itself (I hope my english is sufficient to express things right/understandable in the following), and not to think primarily about the binary tree.

If we start thinking about

− the property of infinitude of that mental object, that we create by counting, which means here the idea of an infinite set of numbers, which is thought to satisfy the requirement, that −unboundedly− for any element n there is another element n+1, (different from n, call n and n+1 neighbours),

− and on the other hand start studying the properties of another infinity, where neighbours cannot be identified, such as the idea of a continuum (*1, and see further remarks below),

then we might assign to those types of infinities a "name", a nominal category.

If we further think, we are able even to put that types of infinities (and maybe some more) in an ordered system, we even may assign them a numerical index instead of a "name". Now this numerical index appears as a number, but is basically an index only establishing an order.

Next step one can study, whether the assigned index−value

correlates with the properties of the different types
of infinity in such a way, that even arithmetical operations in
them make sense (how it is done historically). But before
adding "alephs" or creating thoughts like "2^aleph_1"
I guess it is important to consider your remark (or such like):

> This property of a set is expessed by or as a transfinite number. (In
> German: transfinite Zahl)

the introduction of the word "number" in this context, (even when
specified as "transfinite number") may be a source of confusion.

I always understood "transfinite number" not being a "count−of−elements"
(which would be impossible with uncountable sets) but being an index of
the *type* of infinity (already in mind, that that types have
somehow an order).
That this index is assumed to agree with numerical operations on
it, prominently in its extended interpretation as a size of finite
sets, but also with its interpretation for types of infinite sets
(aleph_1, Aleph_2,...) may be the reason to call it "transfinite
number" instead of, for instance "infinity type index", but that
may be unfortunately misleading, if not always deliberately considered.

It is (different to the "elements" of the set of numbers) a "property"
and not just another element, just another "number" and hence not
comparable to that of the elements of the set.

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Put it in slightly other words:

If we establish a "dimension", which covers

− from the first, naive infinity (the infinity of our
mental object of the natural counting)

− up to that type of infinity, which we assume to be a
"continuum"(*1), (where we cannot single out neighbours),

then the use of the concept of "number of elements" for all
of these sets is in itself misleading. It focuses the mind
to a property, which is not given for the continuum (since
we cannot single out neighbours). So −maybe− prominently
the use of that conception in questions like:

"what is the number of elements of this infinite set
compared with the number of that other infinite set"

is surely an important source of miscommunication about the
more basic (and yet more interesting) properties of comparisions
of types of infinities.

One aspect is, to give it another name, a terminus technicus,
like "cardinality" ("Mächtigkeit" german), to overcome the
connotation of "numbering","counting" (which fixes oneself
to only one type of infinities, as I suggested above), and to make
it clear, that the symbols, which denote different cardinalities
(aleph_xy), are essentially not numbers, but firstly *indices*.

For instance, that following statement induces confusion:

> We can compare the number of nodes between level 0 and level n with the
> number of all nodes.

in the −possible− case, that your tree represents a rule to describe
a continuum(*1): since then it is referring to a "number of all nodes",
which cannot be given, if it possibly is "un−countable".

The concept "number of" may be assigned to describe an (important)
property of a finite set, but if we deal with mathematical objects,
which are constructed to be infinite, it is better to switch to the
terminus "cardinality" (or maybe a better one), to not apriori throw
away the ability to include uncountable(*1) infinities in our
considerations.

That may be an amateurish view of things; it again may mix the
non−neighbour−property of rational numbers with that of real
numbers, and thus is not sufficient to characterize continuum,
but I think it is at least a necessary criterion (and it may
be more helpful after that to line out another, more powerful
distinction)

Gottfried Helms

*1: "Uncountable", for instance "continuum", where we cannot specify
a number and its neighbour, which, for instance occurs by the
introduction of a rule, that we allow a third number always between
two numbers, for instance by the non−bounded application of the
computation of a mean.
If we have such a continuum, we cannot "count" its elements.
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- ***Follow−Ups***:
  - ***Re: Cantor and the binary tree***
    - *From:* Tony Orlow
  - ***Re: Cantor and the binary tree***
    - *From:* mueckenh

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