Re: Topological space, descending chains of closed subsets.,

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- *From*: William Elliot <marsh@xxxxxxxxxxxxxxxxxx>
- *Date*: Sun, 12 Jun 2005 23:36:37 −0700

−−−− From: James <James545@xxxxxxxxx>
Newsgroups: sci.math
Subject: Topological space, descending chains of closed subsets.,

> Dear all, I am having a bit of trouble with this one
> and I was hoping someone could help :

James, did you bother to read my (mars) early reply in the

Ask−a−topologist forum
http://at.yorku.ca/cgi−bin/bbqa?forum=ask_a_topologist&task=list

or perhaps
http://at.yorku.ca/cgi−bin/bbqa?forum=homework_help&task=list

where I conclude basically the same as the others, giving some detail and
show non−uniquiness of decomposition except for Hausdorff spaces?

−− copy of reply
From: James
Subject: Descending chains of closed subsets

> Let X be a topological space such that every descending
> chain of closed sets is eventually constant.

That's some confusing to interpret. I conclude it equivalent to
.. . If C = { Kj | j in I } is a nested family of closed sets,
.. . . . then /_j Kj in C

> Irreducible means a set such that if it is the union of
> two closed subsets F and G, then it equals F or it equals G.

A is irreducible iff A is hyper−connected subspace.

> Show that X can be expressed as a finite union
> X = Y_1 \/ ... \/ Y_n where the Y_i are closed and irreducible
> and Y_i is not a subset of Y_j for any i not equal to j.

If X is Hausdorff, then it's finite and \/{ {x} | x in X }
is the unique decomposition into irreduciables.

Let K0 = X be infinite closed set and from closed infinite Kj
construct closed infinite K_(j−1) as follows. For x /= y in Kj,
some open U,V seperate x,y. Thus Kj\U and Kj\V are proper closed
subsets of Kj. Let K_(j−1) be either those two that is infinite.

This descending sequence of closed sets isn't eventually constant.
Indeed, if K = /_j Kj in C, then some j with K = Kj.
Hence K subset K_(j−1) proper subset Kj = K, which cannot be.

−− in general
If X is irreduciable, then X is the decomposition. Otherwise
some closed A,B with X = A \/ B and neither containing the other.
If A and B are irreduciable, A \/ B is the decomposition.
Otherwise do the same again with A and/or B as needed.
Show this process ends in a finite number of steps.
If perchance, any one of these generated sets contains another,
the remove the smaller. Now you have a finite decomposition.

> Also can you show that this decomposition is unique?

No, it is not unique. For example any finite, included point
topology is counter example. Finite A with for some a in A,
.. . U is open iff a in U or U = nulset.
where the only closed set containing a is A.

{ a,b } decomposes into { a,b } and {a} \/ {b}
{ a,b,c } decomposes into { a,b,c }, {a} \/ {b} \/ {c},
.. . { a,b } \/ {c}, and { a,c } \/ {b}

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- ***References***:
  - ***Topological space, descending chains of closed subsets.,***
    - *From:* James

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