

# Re: roots of polynomials on a field

---

*Source:* <http://sci.tech-archive.net/Archive/sci.math/2005-06/msg03189.html>

---

- *From:* Timothy Murphy <[tim@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx](mailto:tim@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx)>
  - *Date:* Sun, 19 Jun 2005 12:39:21 +0100
- 

Li Yi wrote:

- > We know that a polynomial of degree  $n$  has at most  $n$  roots in a field.
- >>>From the finite field theory, we know that the splitting field of
- >  $x^{(p^n)}-x$  on  $F(\text{char } F = p)$  has the order of  $p^n$ . The proof says that
- >  $x^{(p^n)}-x$  has  $p^n$  distinct roots on  $F$  and then it says that those roots
- > form a field. The conclusion follows.
- >
- > My question is, why does  $f(x) = x^{(p^n)}-x$  has  $p^n$  roots? I know that
- > its roots are distinct since  $f$  and  $f'$  are coprime.

Because it is of degree  $p^n$ , and a polynomial  $p(x)$  of degree  $N$  over  $k$  has  $N$  roots  
(in the algebraic closure of  $k$ , or in the splitting field of  $p(x)$ ).

--

Timothy Murphy  
e-mail (<80k only>): [tim /at/ birdsnest.maths.tcd.ie](mailto:tim/at/birdsnest.maths.tcd.ie)  
tel: +353-86-2336090, +353-1-2842366  
s-mail: School of Mathematics, Trinity College, Dublin 2, Ireland  
.

---

- *References:*
  - ◆ [roots of polynomials on a field](#)
    - ◇ *From:* Li Yi
- Prev by Date: [Re: Why Fourier and Laplace transforms?](#)
- Next by Date: [Re: The brain is like a surface – a hypersurface – a maximum hypersurface.](#)
- Previous by thread: [roots of polynomials on a field](#)
- Next by thread: [Groups defined by as simple as possible rules... as computer strings?](#)
- Index(es):
  - ◆ [Date](#)
  - ◆ [Thread](#)