Re: differential/algebraic geometry

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- *From*: Jannick Asmus <jannick.news@xxxxxx>
- *Date*: Tue, 21 Jun 2005 21:20:34 +0200

On 21.06.2005 07:05, Jannick Asmus wrote:
> On 21.06.2005 05:43, hale@xxxxxxxxxx wrote:
>
>>Orion wrote:
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>>
>>>Is differential/algebraic geometry and multilinear algebra basically
>>>one and the same?
>>
>>
>>I'm currently reading about algebraic geometry, so I am just a
>>beginner in that area.
>>
>>Dieudonne has written a brief article (but very technical) on
>>the history and development of algebraic geometry which is
>>worth reading. I don't have the reference at hand though.
>>It might give some answers to your question.
>>
>>Of course, differential geometry and algebraic geometry are
>>not basically the same since the former deals with smooth
>>functions on R^n while the latter deals with polynomials
>>on k^n (wher R is reals and k is a field).
>
>
> I would like to go a step further: Algebraic geometry is a geometrical
> extension of commutative algebra where local objects are associated to
> commutative rings and not only (finitely generated) algebras over a
> field k. The latter is certainly one of the classical examples of the
> beginning point of the algebraic theory.
>
>
>>However, I get the feeling that they (or their underlying ideas)
>>are related or are similar. For example, both have a version of
>>the Riemann−Roch theorem that relates the genus g with algebraic
>>properties (in the algebraic geometry case).
>
>
> But this goes certainly back to the existence of a (co−)homology theory
> on the objects in differential and algebraic geometry, resp. In my

> opinion, this does not necessarily imply to some kind of similarity.
>
>
>>Also, both consider the germs of functions at a point. Both
>>consider the tangent space at a point.
>
>
> This is a common conceptual approach on rather different objects which
> appear to be similar in very special cases. Here I am thinking of
> Serre's GAGA (géométrie algébrique et géométrie analytique): an
> equivalence of certain sub−categories of analytical and algebraic
> varieties of the field of complex numbers. It basically refers to
> compact varieties which implies that some cohomological vector spaces
> are finite dimensional.
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>
>>One major feature of differential geometry is that the
>>geometry is considered to be locally Euclidean at a point
>>and that charts are used to globally patch sections together.
>>This part I am not sure about: it also appears that algebraic
>>geometry is considered to be locally affine and that charts are
>>used to globally patch secitons together. If this is true, I
>>think this would provide reasons to consider them both to
>>be related or similar.
>>
>>−− Bill Hale
>>
>
>
> Basically, the two theories share a common principle which is widely
> spread in mathematics. If a theory is wanted to be 'geometrized' then
> you look for standard objects translated to a topological space with
> some additional structure (e.g., the sheaf of continuous functions, of
> differentiable functions). Then a geometrical object is a topological
> space with additional structures which locally look like the standard
> objects.
>
> In differential geometry the standard objects are open sets in some real
> vector space (not necessarily finite dimensional) and in algebraic
> geometry affine schemes stemming from a commutative algebra.
>
> Because of the way of construction of geometric objects, the machinery
> of category theory, universal algebra and (co)homology theory apply. So
> far I agree that the theories appear to be similar. But this argument
> seems to valid for all objects constructed as briefly described above.
>
> But the crucial question is what intrinsic properties of the objects can
> be revealed. If you get down to answer this, you get assertions
> distinguishing geometric theories from each other.
>
> Similarities occur due to equivalences of categories. Mostly they are

> provided by huge theorem which are not obvious at all.
>
> I named Serre with one of his famous theorems (GAGA), so I should forget

Oups !!! I meant that I should *not* forget to name A.G.

> to name one of the 'fathers' of algebraic geometry: Alexandre Grothendieck.
>
>
> Cheers,
> J.
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- ***References***:
  - ***differential/algebraic geometry***
    - *From:* Orion
  - ***Re: differential/algebraic geometry***
    - *From:* hale
  - ***Re: differential/algebraic geometry***
    - *From:* Jannick Asmus

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