Re: ordered pairs/n−tuples as collections of sets

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- *From*: "Stephen J. Herschkorn" <sjherschko@xxxxxxxxxxxx>

- *Date*: Sun, 03 Jul 2005 12:53:29 −0400

Elotemuygrande wrote:

> Suppose you define a 2−tuple this way, which I just found in a book.
> T=(a,b)={{a}, {a,b}}
>
> Is there a way to express T_1 and T_2, the first and second elements of the tuple in terms of standard operations on sets like intersection and union and such instead of the gibberish found on http://en.wikipedia.org/wiki/Ordered_pair ? Yes, the gibberish makes sense to me, but it doesn't feel right saying it that way. The ideas I've come up with so far break down, especially when a=b. The book I'm looking through just says thank God we're not doing it this way in this book and no detailed explanation. I'm also thinking about how to define n−tuples without nasty nested pairs.
>
> Any links or ideas appreciated, this is just arm−chair math thinking.

William Elliot gave some lovely definitions for T1 and T2 in terms of unions and intersections. A much uglier one:
Note that an ordered pair p has one or two elements. Exactly one of these must be a singleton. If p has two elements, one of these must be a doubleton whereof the singleton is a subset. T1(p) is the element of the singleton. If p is a singleton, T2(p) = T1(p). Otherwise, T2(p) is the sole element of the doubleton less than the singleton (set difference).

While nested pairs is one way to define n−tuples, it is more common to define an n−tuple as a function with domain n = {0, 1, 2,..., n−1}. You can't do that immediately for ordered pairs, since the definition of function uses ordered pairs itself. Thus, formally, ordered pairs and 2−tuples are different things, but there is a natural correspondence.

I find the Wikipedia article quite clear, but I already know this stuff.

It is not necessarily a good source for someone learning these concepts for the first time. I strongly urge you to pick up a copy of Halmos's Naive Set Theory. You will find a better explanation of such things there, in less compact form, than anywhere on the 'net. I feel all students of mathematics should read it.

Best regards,

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