Re: polynomial for 2^(1/3)−i

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SJ Roh wrote:

      This is a (variant of a) problem in Fraleigh's Abstract
      Algebra: Show that the number
      a = 2^(1/3)-i
      is algebraic over Q by finding f(x) in Q[x] s.t. f(a)=0.
      ( i denotes the square root of -1.)


      According to the book,  the polynomial
      f(x) = x^6 + 3x^4 - 4x^3 + 3x^2 + 12x + 5
      satisfies f(a)=0.


      But how do we find this polynomial f(x)?
      One way I can think of is, write 1, a, a^2, a^3, a^4, a^5, a^6
      as rationally linear combinations of 1, 2^(1/3), 2^(2/3), i,
      i2^(1/3), i2^(2/3) and do some linear algebra to determine the
      coefficients c_0, c_1, ... c_6 satisfying c_0 + c_1 a + c_2
      a^2 +...+ c_6 a^6 = 0.
      I wonder if this is the most usual way to find f(x).
      Is there a less tedious way to find f(x)?


  Let's start with a = 2^(1/3)-i and observe that the conjugate value
a' =2^(1/3)+i must be solution too.


 Let's eliminate the cubic root and note that 'a' must be solution of
(x+i)^3-2 = 0 and a' of (x-i)^3-2 = 0 so that a and a' will be solutions
of the product : ((x+i)^3-2) ((x-i)^3-2) = 0 (your f(x)=0 !)



      For example, if a = 2^(1/2)+3^(1/2),
      we square both sides to get
      a^2 = 5 + 2 6^(1/2), i.e. a^2-5 = 2 6^(1/2),
      and we square again and we get a polynomial f(x) having
      2^(1/2)+3^(1/2) as a root.
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Yes, or as previously write (x−2^(1/2))^2−3 = 0 square, put the sqrt(2) term alone and square again.

Hoping it helped,
Raymond

Thanks in advance.

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