Re: Relative Cardinality

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- *From*: "Proginoskes" <proginoskes@xxxxxxxxxxxxx>
- *Date*: 17 Jul 2005 20:44:38 −0700

mueckenh@xxxxxxxxxxxxxxxxx wrote:
> Proginoskes wrote:
>
> > In order to even _talk_ about the notation 0.333..., you need to have
> > the concept of limits and sums of an arbitrarily large number of
> > numbers.
>
> No problem with potential infinity. I can reproduce an abbreviation for
> a number larger than any one you tell me.

Okay, here we go. You've agreed that the set of all "realizable" numbers (i.e., Muecken numbers) is some fixed finite set, so I choose my number H to be the largest number in that set. (BTW, Since the set is finite, H cannot be infinity.) By definition, you can do no better.

> Knuths arrow−notation or
> Steinhaus' squares are but one way to achieve that.

But you will need an arbitrarily large number of arrows (or squares) to do this, and you don't have that; the set of all Muecken numbers is finite, so you can only do arrow notation or squaring a finite number of times, resulting in a finite number (no matter how large it may be).

> I have a limit,
> namely there is always a (decadic) terminating rational fraction a
> such that | a − 1/3 | < eps for any eps > 0 which can be expressed.

There is no such number a, because | a − 1/3 | is positive (it can't be zero, because there is no _decimal_ terminating fraction for 1/3), and I simply choose eps to be | a − 1/3 | / 2.

You don't seem to realize that you suffer from "quantifier dislexia", that is, in your statement, a is chosen before epsilon. In the true definition of a limit, epsilon is chosen before a.

To see that this order is important, consider the following statements (in the traditional real number system):

* There is a real number x such that, for all real numbers y, x > y.
(This is false.)

* For all real numbers y, there is a real number x such that, x > y.
(This is true; take x = y + 1, for instance.)

> > Since there are only a finite number of Muecken numbers, this
> > means 0.333... is undefined in your system, so you are not allowed to
> > use it.
>
> No. Further there is the possibility to change to the 3−adic system
> where 1/3 = 0.1.

So there are no irrational Muecken numbers, since irrational numbers
cannot be written as a terminating 3−_ary_ decimal.

> > What do you mean by "a simple rule"? Give me a concrete definition.
>
> Consuming less information than 10^100 bits can store.

How is the information encoded?

> > Well, there goes set theory (which says, for one thing, that if S is a
> > set with n elements, the set of all subsets of S has 2^n elements).
>
> The power set of the set of all sets does not exist.

The set of all sets does not exist, even in standard set theory. (It
leads to the contradiction that it equals its own power set.) It's
called a _class_.

> A set of 10^100
> elements is the set of all. It is a bit smaller than Cantor had
> expected but he knew already that not every collection is a consistent
> set.

It is more than a bit smaller than post people have been working with,
as well.

> > > 0.333... may have infinitely many digits 3.
> >
> > _How many_ digits? Infinity is not a number. (Not even in the standard
> > set of real numbers.)
> >
> More than anyone can say. Potential infinity.

How do you express "potential infinity" in the universe?

> > > But the number of different numbers like 1/3 is less than 10^100.
> >
> > What do you mean by "numbers like 1/3"? (Your comment is unnecessary,

> > since the number of Muecken numbers itself has at most 10^100 elements.
> > Adding more conditions to the set only reduces the number of elements.)
>
> We are in the comfortable situation that, contrary to oil, food, and
> water, the bits have not yet become scarce.

But, as I've mentioned, you still only have a FINITE amount of storage.
This means there are certain things which you are unable to do (which
is why not every real number is a Muecken number).

Also, reading back on my comment, I was only saying that the number of
elements in A intersected B is at most the number of elements of A.

> >
> > Then define it. If there's no definition, it's worthless from a
> > mathematical point of view (and you may as well move your posts to
> > alt.fan.numerology).
>
> I have already defined: A natural number exists if there is a
> fundamental set or if there is an n−adic representation realized.

What is a "fundamental set"? When is a number "realized"? Is e
"realizable"?

> > This is not true. You are confusing a property of the set with a
> > property of its elements. The _set_ {Berlin, 23, Mars} is finite, but
> > you'd (presumably) never say that the _element_ Berlin is finite.
> >
> > The _set_ {1, 2, 3, ...} in standard number theory is _infinite_, and
> > its _elements_ are all _finite_.
>
> That example does not fit the problem.

No, it fits your problem precisely. You are confusing the properties of
a set with the properties of the elements. I showed you this in a
fundamental way, as an analogy. If you don't understand the analogy,
you need to do some off−line thinking for a while.

> N consists of elements which, contrary to Berlin,
> 1) are ordered
> 2) count their initial segment.

Your two properties are only further properties that _can_ be assigned
to the set; they are not _necessary_ for the set itself.

I could just as well order the elements of {Berlin, 23, Mars}, by
saying that Berlin < 23 < Mars (and thus Berlin < Mars). And for all
you know, this set _could" "count its initial segment." (Maybe it does
in another language.)

> >

> > Infinite means, literally, not finite.
>
>
> Yes. But not actually infinite.

Infinite means not finite. As in the number of prime numbers, or the number of rational numbers, or the number of points in the plane. None of these sets is finite, so each of these sets is infinite. Period.
There is no "actually" about it.

(When you say a set is "potentially infinite", I am understanding this to mean that there is no upper bound on how big the numbers can actually be. The term for this situation in standard mathematics is "unbounded".)

> Actual infinity is a number (a whole number according to Cantor).

In some systems, yes, it is allowed to be a number. But when dealing with integers, rational numbers, real numbers, complex numbers, infinity is NOT allowed to be a number. So for the purposes here, no real number, and no natural number, can equal infinity, ever.

(In the surreal numbers, or the ordinals, infinity IS allowed to be a number, and it has certain necessary properties. But that's not the case here.)

> Now draw the graph of a simple function f(n) = n where n represents the
> natural numbers and f(n) the numbers {1,2,3,...,n}. This function is
> the diagonal of the first quadrant of the Cartesian co−ordinate system.

(To get that, you would have to let n be any _real_ number. That's not important right now, though.)

> As long as infinity is not actually reached by n it is not reached by
> f(n) and vice versa.

So you would say that this set of points is "potentially infinite", but not "actually infinite", right?

> > Knowing all the digits of the real numbers can be done with arithmetic
> > (the standard long division algorithm, where you find the integer part
> > of the first number, the integer part of the second number, and quit if
> > they're not equal; otherwise, find the tenth digit of the first number
> > and the tenth digit of the second number, and if they're not equal, you
> > know which one is bigger; etc. If you have two different real numbers,
> > this procedure will terminate). This means that real numbers, as
> > defined in the standard real number system, exist.
>
> But it need not terminate before your means of writing down the
> integers is exhausted (including your personal memory).
>

> > which means my point is valid; you do _not_ know all the digits
> > 0.111... unless
> > you allow arbitrarily large numbers to exist. (I've also warned you
> > about the notation 0.111..., which in your number system is
> > meaningless, haven't I? I think so.)
>
> It is not meaningless. There are arbitrary many 1's.

Your set of numbers does not allow the concept of "arbitrarily many" objects; you can only count up to H, where H is the largest Muecken number. "Arbitrarily many" means that for any number N, there are more than N objects. So you can only say that there are more than H 1's, so you can't tell the difference between a string of H+1 of them or an "infinite" number of them.

> The limit is 1/9
> because potential infinity is always sufficient for analysis.

But there's no way for you to distinguish 0.111... from 0.111...111 (where there are H + 1 1's, H being the largest Muecken number).

> > Okay, let's refine the question: You have a list of particles in the
> > universe in some order. You show up "at work", with all the 1's on the
> > particles, in the same order you've established. What number does the
> > universe represent?
>
> You show up at work in front of your screen. It is filled with 1's.
> What number does the screen represent?

You're supposed to answer my question first. However, I will tell you what I would say: It depends on what representation is supposed to be used.

But you can't say this, because you have to have some "rule" which decides this. And this rule will take one configuration of the universe and give you a unique number; there is no possibility for more than one interpretation.

> You need not write down all the 1's of the prime number 700

700 is not prime, not in any base.

> of the form
> 111...111. But it is sufficient to encode the information: All digits
> are 1, the number of digits is 10^25.

But what if 10^25 isn't representable? This encoding won't work in that case.

> As i have written down here. Only
> for numbers which do not follow any simple rule, like pi, we must write

> down all the digits.

What about e−1 ? How do you express that number?

> > There is actually a bit of regression here, once we talk about rules,
> > because the rules have to be interpretted. (The particles that have
> > 111...111 on them also include the particles that tell what the rules
> > are.) So you will need to say how to interpret 111..111 (on the "rule"
> > particles only) first.
>
> That is not a particular problem of mine. It is the same with
> mainstream mathematics.

In mainstream mathematics, there is an established difference between
the number (the object) and the mathematician (the observer). In your
physical description, these two are the same thing, since you're using
every particle in the universe. This is the difference between the two,
and why mainstream mathematics doesn't really have this problem.

> >
> > All of this boils down to the following: There has to be some rule
> > which is not written down. This rule should give you a unique
> > interpretation for every possible representation, which may be simple
> > or complicated, but you still end up with only 10^100 Muecken numbers.
> >
> > If the unwritten (literally 8−)) rule changes, the numbers you get can
> > change, but how do you know which unwritten rule to use, unless it's
> > written down on some particle? And once again we find that the number
> > that the universe represents depends on some rule about rules (which
> > rule do we use?) and what's written on particle X, which may as well
> > have been stated as an unwritten rule in the first place.
>
> That is all the same with any representation of numbers.

This means that there is one specific fixed rule which says how numbers
are encoded, and that variations are not allowed. Your example above
for encoding 111...111 (where there are 10^25) shows that it cannot be
expressed as a "shortcut" ("10^25 1's"), but with the same rule as
every other number (such as the binary system, or base 10 system).

> > How do you know consciousness in physically inside my head? Maybe it
> > "leaks in" from some hyperspace. (This is an unsettled problem
> > discussed by philosophers for centures, so I don't really expect an
> > answer. 8−))
>
> If you are a convinced solipsist you should never attend meetings.

I never said I was a solipsist; I only said that the source of
consciousness might be outside the physical universe. I didn't rule out
the possibility of other consciousnesses.

Don't put words in other people's mouths.

> >
> > Geometry works with what are called "ideal objects": lines that have no
> > width, points that have no dimension, etc. These objects do not exist
> > in the physical world, so mathematics cannot be a part of physics.
>
> Reality supplies the stuff. A bit of abstraction is fine. But I am
> sure, there is no ideal circle or rectangle in your head.

Not in my head, but it might be in my mind (which need not be
restricted to the physical universe).

> >
> > Yes, they can; I can perform an algorithm to find any digit of sqrt(2)
> > in finite time.
>
> Then try to find out and store the first 10^100 digits.
>
> >
> > It is one possible measure for sets. Your "cardinality" is similar, in
> > that it is one possible measure for sets as well. However, you should
> > not have said that your "cardinality" is the same as traditional
> > cardinality, unless you can show that, for any set S, the cardinality
> > of S is the same as the "cardinality" of S. Only then would you have
> > had the right to say your "cardinality" is the well−established
> > cardinality.
>
> Cardinality was introduced as a measure of the number of elements. And
> when introducing it, Cantor mentioned something like clar thought. That
> is also the foundation of my definition.

But your "clear thought" definition has different properties than
Cantor's definition, and should not be treated the same way, unless you
know it results in the same concept. Re−read my "apple/banana" example.

> >
> > In an example which is probably clearer, I could decide to start
> > calling an apple a banana. There's nothing wrong with this, until I
> > start talking with other people about fruit. There, my statements that
> > "some bananas are red" and "bananas are nearly round" make no sense,
> > even though they are perfectly okay from my point of view. This
> > confusion results from using standard terminology in a non−standard
> > way.
> >
> > > Cantor defined it by "our thinking".
> > > He then used bijections to measure it but did not explain why.
> >
> > That's the _definition_ of standard cardinality. It's just like saying
> > that 3 is an abbreviation for 1 + 1 + 1. Why is "3" defined this way?
> > You tell me, and you'll understand why Cantor used bijections.

>
> I have proved tha there are not more irrational numbers than rational
> numbers.

.... if counted in a certain way. If you limit the size of sets to 0, 1,
2, 3, ..., and "infinity" (not divided up any way), then, yes, your
result is true. But you have no right to call your set−measuring
function cardinality unless that's what it actually is. (And since it
isn't, then you should have used another name.)

> >
> > Which in particular don't exist? In particular, what is the smallest
> > value of N such that 1, 2, 3, ..., N are numbers, but N+1 isn't?
>
> How far does our atmosphere stretch out into the space? Please answer
> in millimeters.

I'm asking you to use your definition (which I don't know) to find the
N with the property above, which is not the same as an exercise in
fuzzy logic (which is what you're asking me −− the electron of a
hydrogen atom can be arbitrarily far from the nucleus). If there are
only a finite number of "realizable" numbers, then my question is a
legitimate question: what determines whether a number is "realizable"?
If you can't answer that, then your definition is meaningless. Maybe I
should have asked:

Is 1 a "realizable" number? Why or why not?
Is 2 a "realizable" number? Why or why not?
(etc.)

> >
> > > > When real numbers were defined (based on sequences of rational
> > > > numbers), great care was taken to show when two of these resulting real
> > > > numbers are the same and when they are different.
> > >
> > > In order to define pi by a sequence there is the necessity to show that
> > >
> > > | a_n − pi | < epsilon
> > > for any positive epsilon.
> >
> > (You've mixed up quantifiers here. Pi can be defined by a sequence of
> > rational numbers a_n, but the condition on this sequence is that for
> > every positive epsilon, THERE EXISTS AN N such that |a_n − pi| <
> > epsilon when n >= N. The value of N changes when epsilon changes; you
> > do not need to have |a_n − pi| < epsilon for all n and for all epsilon.
>
> You misunderstood me. Perhaps my fault. It must be possible to satisfy
> this inequality for arbitrarily small epsilon. But already for the
> comparatively large eps = 1/10^10^100 this inequality cannot be
> satisfied by any a_n.

First of all, you're talking about real numbers, so the standard definitions of what numbers exist apply. ("Real number" and "integer" are the standard definitions, not yours.)

For a sequence a_n to converge to pi, it's not required that | a_n − pi | be less than 1/10^10^100 for ALL n, but only for LARGE n. (If we wanted a_n − pi to be small for all n, then the only sequences which would "converge" would be constant sequences, where a_1 = a_2 = a_3 = ...)

If a_n is defined by:

a_1 = 3.1
a_2 = 3.14
a_3 = 3.141
a_4 = 3.1415
....

then a_n is pi rounded to the nth decimal place, which makes
| a_n − pi | <= 1/10^n.

So I can guarantee | a_n − pi | < epsilon = 1/10^10^100 if I insist that n >= 10^100 (maybe plus one).

One thing that I've wondered about (which no one seems to have touched upon) is how you do any calculation in your system of numbers. If the universe can hold only one number at a time, then how do you do anything like addition of two Muecken numbers?

−−− Christopher Heckman

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- ***Follow−Ups***:
    - ***Re: Relative Cardinality***
        - *From:* mueckenh
- ***References***:
    - ***Re: Relative Cardinality***
        - *From:* mueckenh
    - ***Re: Relative Cardinality***
        - *From:* Dik T. Winter
    - ***Re: Relative Cardinality***
        - *From:* mueckenh
    - ***Re: Relative Cardinality***
        - *From:* Randy Poe
    - ***Re: Relative Cardinality***