

## Re: set of a set etc.

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*Source:* <http://sci.tech-archive.net/Archive/sci.math/2005-07/msg03357.html>

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- *From:* "Mark Nudelman" <[markn@xxxxxxxxxxxxxxxxxxxxxxxxxxxx](mailto:markn@xxxxxxxxxxxxxxxxxxxxxxxxxxxx)>
  - *Date:* Wed, 20 Jul 2005 22:16:28 -0700
- 

Jasper wrote:

> Mark Nudelman wrote:

>> Stephen J. Herschkorn wrote:

>>> Jasper wrote:

>>>>

>>>> The description is what I would call formal, not conceptual. "My  
>>>> cat" and the set of my cat {My cat} are different conceptually. My  
>>>> cat likes milk. The "set of my cat" does not, yet the two  
>>>> denotations are closely related. What is the conceptual  
>>>> relationship between the two?

>>> Your cat is a member of the set of your cat. The set of your cat is  
>>> not a member of your cat.

>>>>

>>>> Sets are collections. A collection is distinct from the objects  
>>>> therein (usually). Put a ring in a box. The box contains the ring;  
>>>> the box and the ring are not the same thing.

>>>>>

>>>>> Just to confuse matters, W.V.O. Quine in "Set Theory and Its Logic"  
>>>>> defines the law of extensionality and notes that a consequence of it  
>>>>> is that there is only one memberless object. That is, since  
>>>>> extensionality says that two things are identical if they have the  
>>>>> same members, and individuals do not have members, all individuals are  
>>>>> identical to the empty set and to each other. To avoid this, he  
>>>>> could treat an individual as a different sort of object than a set,  
>>>>> but instead he defines " $x \in y$ " as meaning " $x = y$ " when  $y$  is an  
>>>>> individual. A consequence of this is that individuals are identical  
>>>>> to their unit sets, that is,  $x = \{x\}$  but ONLY when  $x$  is an  
>>>>> individual. Of course, he retains  $x \neq \{x\}$  when  $x$  is a set. He  
>>>>> takes some pains to show why this is harmless, but it does seem  
>>>>> rather odd.

>>>>>>

>>>>>> --Mark

>>>>>>>

>>>>>>> Yes it does. Thanks for the input and the reference. What do you make  
>>>>>>> of it?

Well, Quine discusses other possible solutions. He mentions using two different styles of variables, one for individuals and one for sets, which is I think the most natural approach. This is probably what many of your

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respondents have in mind when they point out that a cat is not the same as the set of a cat, etc. (Otherwise, they'd be forced to conclude that a cat is the same as the null set, since neither has any members [potential jokes about tom cats at this point notwithstanding].) Quine also mentions the possibility of adding a predicate that asserts "individuality", or conversely, "classitude", which would let us distinguish individuals from sets. But he prefers his solution as more elegant, since it doesn't require an extra predicate or separate variable styles. I quote from Quine:

We are interested in " $x \in y$ " to begin with only for classes  $y$ ; such are the only cases of " $x \in y$ " that are subject to preconceptions worth respecting. If for the sake of smooth systematization we see fit to assign meaning to further cases, let us assign a meaning that maximizes the smoothness.... Let us rule " $x \in y$ " true or false according as  $x = y$  or  $x \neq y$ , when  $y$  is an individual.... But what if  $y$  is an individual and  $z$  is the unit class of  $y$ ? On our new interpretation ... " $x \in y$ " then becomes true if and only if  $x$  is the individual  $y$ ; so  $(\forall x)(x \in y \text{ iff } x \in z)$  and therefore  $y = z$ . This result is prima facie unacceptable, since  $y$  is an individual and  $z$  is a class. But actually it is a harmless result; none of the utility of class theory is impaired by counting an individual, its unit class, the unit class of that unit class, and so on, as one and the same thing. True, we are well advised now to adjust our terminology to the extent of ceasing to explain "individual" as "nonclass"; let us take to saying that what constitutes them individuals is not inclassitude, but identity with their unit classes.... Everything comes to count as a class; still, individuals remain marked off from other classes in being their own sole members.

---End of quote

The last point is a key one I think -- by this route, everything is a class to Quine, which simplifies some things. It's important to keep in mind that this is just the way Quine's axioms work. Many (probably most) other axiomatic systems don't consider  $x = \{x\}$  to be true, even if  $x$  is an individual. So there's no one true answer to the question about what this means, it depends on the axiom system you're using. But there's no argument when talking about multi-element sets: for Quine as for everyone else, the set  $\{x,y\}$  is different from the set  $\{\{x,y\}\}$ , since the first has two elements, and the second has one element.

--Mark

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• *Follow-Ups:*

- ◆ *Re: set of a set etc.*  
◇ *From: k wallace*
- ◆ *Re: set of a set etc.*  
◇ *From: Jasper*

Re: set of a set etc.

• **References:**

- ◆ **set of a set etc.**  
◇ From: Jasper
- ◆ **Re: set of a set etc.**  
◇ From: Jean-Claude Arbaut
- ◆ **Re: set of a set etc.**  
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- ◆ **Re: set of a set etc.**  
◇ From: Jasper
- ◆ **Re: set of a set etc.**  
◇ From: G . Frege
- ◆ **Re: set of a set etc.**  
◇ From: William Elliot
- ◆ **Re: set of a set etc.**  
◇ From: Jasper
- ◆ **Re: set of a set etc.**  
◇ From: Dave Seaman
- ◆ **Re: set of a set etc.**  
◇ From: Jasper
- ◆ **Re: set of a set etc.**  
◇ From: Stephen J. Herschkorn
- ◆ **Re: set of a set etc.**  
◇ From: Mark Nudelman
- ◆ **Re: set of a set etc.**  
◇ From: Jasper

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