Self Study problem help − Group theory

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- *Date*: 25 Jul 2005 18:00:24 −0700

I am studying the third edition of 'Abstract Algebra' by Dummit and Foote on my own and have come across a question which I not certain I have solved correctly, or rigorously. As always even though this is self study I would ask that you give as little help as possible and avoid giving me an answer, only guideance on how to improve my proof, or where the reasoning went wrong rather then an explicit solution.

The problem is (paraphased) 'Exhibit a proper subgroup of the rationals under addition which is not cyclic'.

I had trouble at first, but I think I have a good answer, I'm just not certain how rigorous it is. This text defined a cyclic group to be a group that's generated by a single element.

So, consider X which is generated by the infinite collection <1/p_1, 1/p_2, ...> where p_n is a prime greater then 2. Now note that adding any two two together gives the product (p_i + p_k)/(p_i * p_k). So, all the elements generated by this collection will have an integer for a numerator and the denominator will decompose into powers of the primes p_n listed in the numerators of X. Now, no matter what the numerator is, when we reduce the fraction we can only divide out by the primes in the bottom, and none of them are 2, so no matter what we can never generate 1/2. (I'm not certain if my reasoning is solid here, I keep thinking somehow I can create 1/2 by a clever combination of additions and subtractions on the rationals I have). Now this means we have a proper subset, since we chose to generate the group we know that it's closed.

Now assume that X is cyclic. This would mean some element, well call it k = (v / [(p_a)^x*(p_b)^y*...(p_c)^z] would generate X. But, since k^m for all integers m would only contain powers of a finite number of prime denominators there would exist elements of X that k would not construct. This contradicts the assumtion that X is cyclic. This part of the "proof" relies on the same logic about the denominator being a product of primes so I feel if the first part fails this part will to.

Thanks in advance, Abe Buckingham.

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    - *From:* Gerry