

Re: Self Study problem help – Group theory

Source: <http://sci.tech–archive.net/Archive/sci.math/2005–07/msg04039.html>

- *From:* Gerry Myerson <gerry@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>
 - *Date:* Tue, 26 Jul 2005 11:49:02 +1000
-

In article <1122339624.677755.300860@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>, abe.buckingham@xxxxxxxxxx wrote:

You've pretty nearly got it right. See below for a comment or two.

- > I am studying the third edition of 'Abstract Algebra' by Dummit and
- > Foote on my own and have come across a question which I not certain I
- > have solved correctly, or rigorously. As always even though this is
- > self study I would ask that you give as little help as possible and
- > avoid giving me an answer, only guidance on how to improve my proof,
- > or where the reasoning went wrong rather than an explicit solution.
- >
- > The problem is (paraphased) 'Exhibit a proper subgroup of the rationals
- > under addition which is not cyclic'.
- >
- > I had trouble at first, but I think I have a good answer, I'm just not
- > certain how rigorous it is. This text defined a cyclic group to be a
- > group that's generated by a single element.
- >
- > So, consider X which is generated by the infinite collection <1/p₁,
- > 1/p₂, ...> where p_n is a prime greater than 2. Now note that adding
- > any two together gives the product (p_i + p_k)/(p_i * p_k). So, all
- > the elements generated by this collection will have an integer for a
- > numerator and the denominator will decompose into powers of the primes
- > p_n listed in the numerators of X.

Where do the *powers* of primes come from? Is 1/9 in X?

- > Now, no matter what the numerator
- > is, when we reduce the fraction we can only divide out by the primes in
- > the bottom, and none of them are 2, so no matter what we can never
- > generate 1/2. (I'm not certain if my reasoning is solid here, I keep
- > thinking somehow I can create 1/2 by a clever combination of additions
- > and subtractions on the rationals I have).

You should be able to prove that if a, b, c, d are integers with b, d odd then the denominator of (a/b) + (c/d), in lowest terms, is odd.

Re: Self Study problem help – Group theory

- > Now this means we have a
- > proper subset, since we chose to generate the group we know that it's
- > closed.
- >
- > Now assume that X is cyclic. This would mean some element, well call it
- > $k = (v / [(p_a)^x * (p_b)^y * \dots * (p_c)^z])$ would generate X. But, since k^m
- > for all integers m would only contain powers of a finite number of
- > prime denominators there would exist elements of X that k would not
- > construct.

The operation being addition, k^m is not germane.

—

Gerry Myerson (gerry@xxxxxxxxxxxxxxxx) (i -> u for email)

.

• **Follow-Ups:**

- ◆ **Re: Self Study problem help – Group theory**
◇ From: abe . buckingham

• **References:**

- ◆ **Self Study problem help – Group theory**
◇ From: abe . buckingham

- Prev by Date: **Re: Exact Trisecting**
- Next by Date: **Re: Four Color Theorem.**
- Previous by thread: **Re: Self Study problem help – Group theory**
- Next by thread: **Re: Self Study problem help – Group theory**
- Index(es):
 - ◆ **Date**
 - ◆ **Thread**