Re: Self Study problem help − Group theory

Source: http://sci.tech−archive.net/Archive/sci.math/2005−07/msg04072.html

---

- *From*: abe.buckingham@xxxxxxxxx
- *Date*: 25 Jul 2005 21:00:20 −0700

Ok let me see if I can get it. I did mean to say that (a/b) + (c/d) had odd denominator naturally, that word seems to be giving me trouble today.

<k> definitely does not contain 1/k^2, it's all the integer multiples of k. I see where I caused the confusion by saying k^m when I didn't mean expodentiation but rather just repeated applications of the group operation, and should have written k*m instead to be less ambiguous. I'm simply accustomed to the text using the multiplicative notation for group operations so I continued with that absent−mindedly without considering my audience can't read my mind − sorry about that.

To answer your new question I can see now how taking the reciprocal of any two distinct primes would have been enough. Just one prime would make a cyclic group (obvious from the definition) so I'd need two. If I took say <1/2, 1/3> I could then take (1/3) + (1/3) + (1/3) to get 1, and − (1/2) − (1/3) = −5/6 therefore 1−(5/6) and make 1/6. Since the denominator is always at most 6 since I can't generate powers of 2 or 3 then 1/6 generates 1/3 and 1/2 it generates the group and it's cyclic, therefore does not meet the requirements.

So any abstracting I can see that given two primes p and q then I can construct all numbers of the form (n*p + m*q)/(p*q) for integers n and m and since p and q are relatively prime there exists an n and m such that n*p + m*q = 1 and therefore 1/(p*q) can be constructed and generates the group. since p/(p*q) = 1/q and q/(p*q) = 1/p.

Now consider a finite collection of primes. We can proceed by induction (?) since we know that for any two we can construct 1/(p*q) which generates that group. Given another prime called say, r we can construct a linear combination of n*(p*q) + m*r = 1 since r and p*q are relatively prime meaning we can construct 1/(p*q*r) and so on with until we exaust the primes on our list since each prime will be relatively prime to the product of the previous primes.

So I'm stuck taking an infinite collection of primes? Seems I would need to rely on the infinitude of primes in order to ensure that no element could generate the rest of them. I could choose a 'sparse' collection of primes though, say every billionth prime and the arguement would hold? This way I can always choose a prime that isn't in the finite number that would be in the denomin