Re: Self Study problem help − Group theory

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- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Tue, 26 Jul 2005 00:45:04 −0700

On 25 Jul 2005 21:00:20 −0700, abe.buckingham@xxxxxxxxx wrote:

>Ok let me see if I can get it. I did mean to say that (a/b) + (c/d) had
>odd denominator naturally, that word seems to be giving me trouble
>today.
>
><k> definitely does not contain 1/k^2, it's all the integer multiples
>of k. I see where I caused the confusion by saying k^m when I didn't
>mean expodentiation but rather just repeated applications of the group
>operation, and should have written k*m instead to be less ambiguous.
>I'm simply accustomed to the text using the multiplicative notation for
>group operations so I continued with that absent−mindedly without
>considering my audience can't read my mind − sorry about that.
>
>To answer your new question I can see now how taking the reciprocal of
>any two distinct primes would have been enough. Just one prime would
>make a cyclic group (obvious from the definition) so I'd need two. If I
>took say <1/2, 1/3> I could then take (1/3) + (1/3) + (1/3) to get 1,
>and − (1/2) − (1/3) = −5/6 therefore 1−(5/6) and make 1/6. Since the
>denominator is always at most 6 since I can't generate powers of 2 or 3
>then 1/6 generates 1/3 and 1/2 it generates the group and it's cyclic,
>therefore does not meet the requirements.
>
>So any abstracting I can see that given two primes p and q then I can
>construct all numbers of the form (n*p + m*q)/(p*q) for integers n and
>m and since p and q are relatively prime there exists an n and m such
>that n*p + m*q = 1 and therefore 1/(p*q) can be constructed and
>generates the group. since p/(p*q) = 1/q and q/(p*q) = 1/p.
>
>Now consider a finite collection of primes. We can proceed by induction
>(?) since we know that for any two we can construct 1/(p*q) which
>generates that group. Given another prime called say, r we can
>construct a linear combination of n*(p*q) + m*r = 1 since r and p*q are
>relatively prime meaning we can construct 1/(p*q*r) and so on with
>until we exaust the primes on our list since each prime will be
>relatively prime to the product of the previous primes.
>
>So I'm stuck taking an infinite collection of primes? Seems I would
>need to rely on the infinitude of primes in order to ensure that no

>element could generate the rest of them. I could choose a 'sparse'
>collection of primes though, say every billionth prime and the
>arguement would hold? This way I can always choose a prime that isn't
>in the finite number that would be in the denominator of the would be
>generator. Or have I missed something important?

Well, yes and no.

Your reasoning is absolutely right that any finitely generated subgroup is cyclic.

So to get a non−cyclic subgroup, we surely need an infinite set of generators.

Nothing wrong with choosing all odd denominators −− it's a nice argument, and doesn't really need improvement. I was just asking about possible variations.

How about using for generators: 1/p where p ranges over any infinite sequence of primes p_1,p_2,p_3, ... with at least prime missing (maybe more, maybe infinitely many). Doesn't that work just as well? Ok, but then using all odd primes is simpler, cleaner −− I agree. I was just exploring a variation.

However there is one clean variation that is worth exploring.

Let p be any prime.

Consider the group <1/p,1/p^2,1/p^3,...>

Here we really do mean exponentiation, although the group is still the additive group generated by those powers. Ok, is that group cyclic?

Based on what you've analyzed so far, paired perhaps with a few more explorations, it seems that you pretty much have the additive group of Q figured out.

quasi
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- ***Follow−Ups***:
  - ***Re: Self Study problem help − Group theory***
    - *From:* abe . buckingham
  - ***Re: Self Study problem help − Group theory***
    - *From:* quasi

- ***References***:
  - ***Self Study problem help − Group theory***
    - *From:* abe . buckingham

- ♦ ***Re: Self Study problem help − Group theory***
  - ◊ *From:* Gerry Myerson
- ♦ ***Re: Self Study problem help − Group theory***
  - ◊ *From:* abe . buckingham
- ♦ ***Re: Self Study problem help − Group theory***
  - ◊ *From:* quasi
- ♦ ***Re: Self Study problem help − Group theory***
  - ◊ *From:* abe . buckingham

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