Hansen chains do not always produce optimum addition chains

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- *Date*: Sun, 31 Jul 2005 22:01:49 −0700

In The Art of Computer Programming, Third Edition, Vol. 2, 4.6.3,
Page 485, question 42 is:
'Is l(2n−1) ? n − 1 − l(n) for all positive integers n? Does equality
always hold? Does l(n) = l0(n)?'.
On Friday night I located a counter example to the last part of this
question. There are indeed some integers with l(n) < l0(n). Using a
branch and bound algorithm described in [1] with substantial additional
pruning of my own I was able to locate the smallest example. I found
that l(5784689) = 27 and there are only two chains if we eliminate
duplicates based on the reduced graphs. This is one of the 16 possible
chains:
1 2 4 8 16 32 64 65 97 128 225 353 706 1412 2824 5648 11296 22592 45184
90368 180736 180801 361537 723074 1446148 2892296 2892393 5784689
This chain has the deviation from l0 at 128.
I recalculated all the chains for 5784689 using a completely different
method described in [2].
I am interested in peoples thoughts on further directions. I am thinking
for example to locate a second value with this property and attempt to
spot a pattern. It would be good if I could mathematically prove the
existence of such chains but I doubt my math skills are strong
enough.
Neill.

[1] A. Flammenkamp and D. Bleichenbacher, 'An Efficient Algorithm for
Computing Shortest Addition Chains', 1997 Discrete Math.
[2] E. G. Thurber, 'Efficient Generation of Minimal Length Addition
chains', SIAM Journal of Computing, Vol. 28, 1999, p. 1247−1263

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- ***Follow−Ups***:
  - ***Re: Hansen chains do not always produce optimum addition chains***
    - *From:* Gerry Myerson

- Prev by Date: ***Re: Cononical Form***
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