

# Re: Random Probability

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> Hi all,  
>  
> I tried to look at 'Coupon collector' but I cannot  
> find anything really  
> comprehensive.. most said it is a classic problem  
> with an obvious  
> result :-p  
>  
> Anyway, What I need is  
>  
> the Probability of getting a complete set on exactly  
> the Nth instance  
> (ie, set was incomplete in the first N-1 instances)  
>  
> Can someone please help to derive and explain a  
> general expression in  
> terms of N and k?  
>  
> Thanks a million!!!  
>

For topics like this you should look up D. Knuth, "The Art of Computer Programming" (more precisely, vol 2, p 64)

Here is the gist of it in my own words. Suppose that the sequence

$a_1 a_2 a_3 \dots a_n$  (\*)

is "complete" (as to occurrences of the numbers 1,...,k), whereas

$a_1 a_2 a_3 \dots a_{(n-1)}$  (\*\*)

is still incomplete. The probability for the latter event is

$S_2(n-1, k-1) k! / k^{(n-1)}$

where  $S_2(n-1, k-1)$  is referring to Stirling numbers of the second kind. In fact,  $S_2(n-1, k-1)$  gives by definition the number of all partitions of an  $(n-1)$ -element set into exactly  $k-1$  subsets, corresponding to the  $k-1$  numbers, which actually occur. Furthermore, those numbers can be arbitrarily permuted in  $(k-1)!$  ways

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and the missing number can be chosen in  $k$  ways, accounting for the factor  $k!$  above. Finally, the total number of sequences of length  $n-1$  is  $k^{n-1}$ , which is the denominator of the fraction above.

The combined probability  $p(n,k)$  that (\*) is complete, while (\*\*) was still incomplete, i.e. the probability you were asking for, is then

$$p(n,k) = \frac{S_2(n-1, k-1)k!}{k^n}$$

because  $a_n$  is the missing number with probability  $1/k$ .

Johann

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- **References:**

- ◆ **Re: Random Probability**

- ◇ From: Bob

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