

# Re: positive definite functions

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- *From:* "Robert Israel" <[israel@xxxxxxxxxxxx](mailto:israel@xxxxxxxxxxxx)>
  - *Date:* 8 Aug 2005 15:24:29 -0700
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David C. Ullrich wrote:

> On 8 Aug 2005 06:41:50 GMT, [israel@xxxxxxxxxxxx](mailto:israel@xxxxxxxxxxxx) (Robert Israel) wrote:  
>  
>> In article <[john.lord-3786F5.16274706082005@xxxxxxxxxxxxxxxx](mailto:john.lord-3786F5.16274706082005@xxxxxxxxxxxxxxxx)>,  
>> john f. lord <[john.lord@xxxxxxxxxxxx](mailto:john.lord@xxxxxxxxxxxx)> wrote:  
>>> In article <[soj6f119lvfg6h7mqg2q4fdmnei2k1aq8@xxxxxxx](mailto:soj6f119lvfg6h7mqg2q4fdmnei2k1aq8@xxxxxxx)>,  
>>> David C. Ullrich <[ullrich@xxxxxxxxxxxxxxxx](mailto:ullrich@xxxxxxxxxxxxxxxx)> wrote:  
>>  
>>>> Anyway, the interesting thing here is Bochner's theorem: A function  
>>>> on R is the Fourier transform of a Borel probability measure \_if and  
>>>> only if\_ it's continuous, positive definite and equals 1 at the  
>>>> origin. (Similarly in more general settings, for example locally  
>>>> compact abelian groups at least.)  
>>  
>>>> Now what my question was meant to be: are there ways to verify (at least  
>>>> for some classes of functions) if a function is positive definite,  
>>>> without calculation of its Fourier transform or of the determinants in  
>>>> the definition of positive-definiteness, as both in general are  
>>>> prohibitively complicated?  
>>  
>>>> In some cases, yes. For example, any "autocorrelation" function of the form  
>>>>  $f(x) = \int_{-\infty}^{\infty} g(t+x) \overline{g(t)} dt$  (where  $g$  is square-integrable)  
>>>> is positive definite.  
>>  
>>>> Given  $f$ , how do you tell whether there exists such a  $g$ ?

In general, you don't (unless by calculating the Fourier transform).

What

I meant was that  $f$  might be initially given in this form, or as a sum of functions of this form.

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- **References:**
  - ◆ **positive definite functions**
    - ◇ *From:* john f. lord
  - ◆ **Re: positive definite functions**
    - ◇ *From:* Dave Rusin
  - ◆ **Re: positive definite functions**
    - ◇ *From:* David C . Ullrich
  - ◆ **Re: positive definite functions**
    - ◇ *From:* john f. lord
  - ◆ **Re: positive definite functions**
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