Re: What does Gödel's Incompleteness mean for the Working Mathematician?

Source: http://sci.tech−archive.net/Archive/sci.math/2005−08/msg01774.html

- *From*: "Bob Stewart" <bobstewart_III@xxxxxxxxxxx>
- *Date*: 10 Aug 2005 07:54:04 −0700

Bhup wrote:
> On Jul 31, 6:37 pm, Bob Stewart wrote in sci.math:
>
> Axiomatic languages do not lead to contradiction unless we have reason
> to believe that either the axioms we have adopted, or the rules of
> inference we have specified, harbour an inconsistency.
>
> There is no reason to believe that either is the case in Peano
> Arithmetic, which is, arguably, amongst the most intuitive of our
> mathematical languages.
>
> So any uncertainty regarding its consistency must reflect our
> interpretation of Goedel's reasoning, rather than the substance of it.
>
> Here is an alternative interpretation which suggests that concerns, of
> the kind that you raised, could be unfounded:
>
> PA is instantiationally complete, but algorithmically incomplete: An
> alternative interpretation of Gödelian incompleteness under Church's
> Thesis that links formal logic and computability
>
> http://alixcomsi.com/PA_is_instantiationally_complete.htm
>
> Intuitively, the thesis of the paper seems quite reasonable: If an
> algorithm computes an arithmetical relation R, treated as a Boolean
> function, as always true, then it must yield a proof sequence for [R]
> in first order Peano Arithmetic.
>
> The consequences are, however, quite drastic for the limitations on our
> understanding of classical mathematics as suggested by standard
> interpretations of classical theory.
>
> On the other hand, the knowledge that every false arithmetic statement
> is disprovable in some particular instance (even though this may take
> an exhaustive search), or instantiationally provable in every instance
> (even if only exhaustively, and not algorithmically), should actually
> invigorate mathematics.

> Although the search for neat, algorithmic, solutions may be
> intellectually satisfying, the search for proofs that differ from case
> to case is certainly more challenging − witness the solution of the
> 4−colour problem − and possibly more rewarding, since the most
> significant of natural phenomena can rarely be seen to be amenable to
> neat, algorithmic, expression and representation within mathematical
> languages.
>
> Regards,
>
> Bhup

Hi Bhup,

Thanks for the reference, I did look at your paper (it is very nicely presented :−)). It struck me that perhaps what you are saying is related to some of Brouwer's intuitionistic theories. I remembered a story that someone had told me about the tragic irony of Brouwer's life and achievements. He was the first person to give a rigorous definition of a topological space (the one that we see in textbooks). He discovered some of the finest theorems in topology (fixed point theorems, theorems about dimension). Yet, later in his life, he rejected his own work. As I understand, in Intuitionism, one rejects the law of the excluded middle. So a statement is not necessarily either true or false. One must define "sets" as consisting of objects that can be actually "constructed". That is, one must be able to construct each object of a set in a finite number of steps (specify an algorithm). However, this leads to rejecting some of the theorems of topology (specifically Brouwer's fixed point theorem in plane topology is no longer valid!). I have not checked any of the statements I have made above, and not being an expert in logic, please correct me if I have said anything wrong!

Cheers,
Bob Stewart

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- ***References***:
  - ♦ ***What does Gödel's Incompleteness mean for the Working Mathematician?***
    - ◊ *From:* Bob Stewart
  - ♦ ***Re: What does Gödel's Incompleteness mean for the Working Mathematician?***
    - ◊ *From:* Bhup

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