Re: infinity

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- *From*: "Jiri Lebl" <jirka@xxxxxx>
- *Date*: 10 Aug 2005 16:46:02 −0700

Kirby Cook wrote:
> Thanks to your patience, I think I follow your arguement. The reason I
> don't buy it lies in my reluctance to accept as sense the concurrent
> statements, 1) the balls in the vase increase infinitely as the time
> approaches noon, 2) There is no point at which all the balls vanish (or
> even any discoverable point at which they begin to be reduced), and 3)
> There are no balls in the vase at noon. That seems to me to be as clear
> a contradiction as contradictions get.

It is only a contradiction to intuition. The reason is that the
problem is stated in terms of real world objects of which we have some
expactations. I'll restate pretty much the same problem in terms of
integration over the real line and then your intuition will not get in
the way:

Let f_n(x) be the characteristic function of the set [n,10n], that is
f_n(x) = 1 if x is in [n,10n] and 0 otherwise. Then the integral of
f_n over the whole real line is equal to 10n−n, that is

int f_n(x) dx = 10n−n = 9n

Note that as n increases this is an increasing set of numbers. Now the
pointwise limit of f_n as n goes to infinity is the zero function as
for a fixed x, f_n(x) will be 0 when n > x. Call
f(x) := lim_{n−>oo} f_n(x). So obviously f(x) = 0 for all x. Now

lim_{n−>oo} int f_n(x) dx = lim_{n−>oo} 9n = oo

but

int lim_{n−>oo} f_n(x) dx = int f(x) dx = int 0 dx = 0

That is, the limit of the integrals is NOT the integral of the limit.

Now let's see why this is the SAME exact problem as the one with the
balls and the vase. The vase is the real line, and the balls
correspond to unit invervals on the real line. Now the functions f_n
could perhaps be thought of as weight distributions of the balls,
though then you'd need rectangular "balls" I suppose and each ball

would have mass 1. To count how many balls are in the vase, you just integrate the function which describes the balls, that is either f_n or f. At time noon − 2^−n we have int f_n(x) dx = 9n balls and at noon we have int f(x) dx = 0 balls. You would have a hard time convincing people that f(x) was anything other then the zero function.

Moral of the story is that not all properties or operations (such as counting the balls or taking an integral) always commute with a limit. That is, not all things in mathematics are "continuous," even though real life and intuition seem continuous.

Jiri

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- ***References***:
  - ♦ ***Re: infinity***
    - ◊ *From:* Theo Jacobs
  - ♦ ***Re: infinity***
    - ◊ *From:* aeo6
  - ♦ ***Re: infinity***
    - ◊ *From:* David R Tribble
  - ♦ ***Re: infinity***
    - ◊ *From:* aeo6
  - ♦ ***Re: infinity***
    - ◊ *From:* David Kastrup
  - ♦ ***Re: infinity***
    - ◊ *From:* aeo6
  - ♦ ***Re: infinity***
    - ◊ *From:* William Hughes
  - ♦ ***Re: infinity***
    - ◊ *From:* aeo6
  - ♦ ***Re: infinity***
    - ◊ *From:* William Hughes
  - ♦ ***Re: infinity***
    - ◊ *From:* Kirby Cook
  - ♦ ***Re: infinity***
    - ◊ *From:* William Hughes
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  - ♦ ***Re: infinity***

- ◊ *From:* William Hughes
- ♦ ***Re: infinity***
  - ◊ *From:* Kirby Cook

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