

$$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C(x_1^2 + x_2^2 + x_3^3 + x_4^2)^{3/2}$$

$$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C(x_1^2 + x_2^2 + x_3^3 + x_4^2)^{3/2}$$

Source: <http://sci.tech-archive.net/Archive/sci.math/2005-08/msg03357.html>

- From: Kira Yamato <kirakun@xxxxxxxxxxxxxxx>
 - Date: Wed, 17 Aug 2005 22:38:01 GMT
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I've having trouble showing for any four real numbers x_1, x_2, x_3, x_4 , there exists a real constant C such that

$$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C(x_1^2 + x_2^2 + x_3^3 + x_4^2)^{3/2}.$$

This is what I got so far: We can reduce it to the case of considering the unit-sphere only. This is because the equation is homogeneous. So, next I try to use lagrange multiplier to find the maximum value of the left-hand side on the unit-sphere. However, the algebra becomes unwielding and I cannot proceed.

Now this was an in-class exam question, and it should not have required more than 15 minutes to solve. Am I missing a trick that I don't see?

I also tried using symmetry argument. The left-side side is symmetry for every pair of coordinates. So, it's intuitive to guess that the maximum occurs at $x_1=x_2=x_3=x_4$. However, I don't know how to justify this rigorously.

Please help. Thanks.

-kira

- Follow-Ups:
 - ◆ [Re: \$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C\(x_1^2 + x_2^2 + x_3^3 + x_4^2\)^{3/2}\$](#)
 ◇ From: The Qurqirish Dragon
 - ◆ [Re: \$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C\(x_1^2 + x_2^2 + x_3^3 + x_4^2\)^{3/2}\$](#)
 ◇ From: quasi
 - ◆ [Re: \$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C\(x_1^2 + x_2^2 + x_3^3 + x_4^2\)^{3/2}\$](#)
 ◇ From: quasi
 - ◆ [Re: \$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C\(x_1^2 + x_2^2 + x_3^3 + x_4^2\)^{3/2}\$](#)
 ◇ From: Peter L. Montgomery

- Prev by Date: [Re: Han's startling new set theory.](#)

$$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C(x_1^2 + x_2^2 + x_3^3 + x_4^2)^{3/2}$$

$$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C(x_1^2 + x_2^2 + x_3^3 + x_4^2)^{3/2}$$

- Next by Date: ***Re: infinity***
- Previous by thread: ***p-adic transcendental numbers***
- Next by thread: ***Re: $x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \leq C(x_1^2 + x_2^2 + x_3^3 + x_4^2)^{3/2}$***
- Index(es):
 - ◆ ***Date***
 - ◆ ***Thread***