Re: Euler's formula for polyhedra

Source: http://sci.tech−archive.net/Archive/sci.math/2005−08/msg04443.html

- *From*: gregegan@xxxxxxxxxxxxxxxxxxxx (Greg Egan)

- *Date*: Wed, 24 Aug 2005 07:34:18 +0800

In article <defnam$44k$1@xxxxxxxxxxxxxxxxxxxxxx>, "George Szpiro"
<george@xxxxxxxxxxxxxxx> wrote:

> Hello everybody:
>
>
>
> A cube has 8 vertices, 12 edges and 6 faces, so the Euler formula v−e+f
> =2 −2g holds, because the genus of the cube is 0.
>
>
>
> Now I bore a square tunnel through the cube. I get v=16, e=24 and f=8 if I
> do not count the sides of the cube where the tunnel exits as faces. (FIRST
> QUESTION: Is this correct?) Euler's formula again holds: v−e+f=0, because
> the genus of a cube with one hole is one.

No, this isn't correct. You have created "annular" faces from the two
square faces with squares punched out of them, and you can neither count
them in the normal way, nor just decide not to count them at all.

The correct thing to do is to divide the two annular faces into four
quadrilaterals each, by joining the vertices of the inner and outer
squares. This will add 8 to the face count and 8 to the edge count that
you quote above, giving a total of v=16, e=32, f=16, so v−e+f=0.

> Now I bore two tunnels through the cube, such that they enter and exit at
> two opposite sides. V=24, e=36, f=10. Euler's formula gives −2, because the
> genus of a cube with two holes is 2.

No, the correct way to deal with each of the tunnels, when they're made
separately through intact faces, is to transform:

v −> v + 8 (the 8 vertices of the tunnel)
e −> e + 20 (the 12 edges of the tunnel, plus 8 dividing edges
for subdividing the annular faces)
f −> f + 10 (the 4 faces of the tunnel, plus 2 x 3 extra faces
when two square faces are subidvided into 4
quadrilaterals each)

The Euler characterestic thus has a net change of 8−20+10 = −2, as you'd expect from increasing its genus by 1.

> NOW MY QUESTION:
>
> If I bore two PARALLEL tunnels through the cube, I get v=24, e=36 and f=12
> and Euler's formula gives 0. But the genus of a cube with two holes should
> be 2?

No, you have to subdivide the two faces (which now have two holes in them) into pieces with single borders. A face with two holes in it has three borders, and you can neither just ignore these, nor count them as if they were normal faces.

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Greg Egan

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- ***Follow−Ups***:
  - ♦ ***Re: Euler's formula for polyhedra***
    - ◊ *From:* Lee Rudolph

- ***References***:
  - ♦ ***Euler's formula for polyhedra***
    - ◊ *From:* George Szpiro

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