# Re: help with diophantine equation

***Source:*** http://sci.tech−archive.net/Archive/sci.math/2005−08/msg04444.html

- *From*: Bill Dubuque <wgd@xxxxxxxxxxxxxxxxxxxx>
- *Date*: 23 Aug 2005 19:27:54 −0400

john_ramsden@xxxxxxxxxxxxxx wrote:
>mechmech wrote:
>>
>> I have this equation: 3*n*n = 3*k*k + 73*k + 14
>>
>> I am interested in the most efficient way to get the solution
>> ( which is n=32 and k=22 ) besides the trial and error method
>
> It's a difference of squares, i.e. after multiplying every term
> by 12 it can be rearranged as:
>
> (6k + 73)^2 − (6n)^2 = 5161
>
> (6(k−n) + 73) (6(k+n) + 73) = 5161
>
> So the most efficient method of solution is to list every
> way in which 5161 can be expressed as the product of two
> factors (including both negative!):
>
> +− 1, +− 5161
> +− 13, +− 397
> +− 397, +− 13
> +− 5161, +− 1
>
> and equate linear factors to each pair and
> see if the result gives integers k and n.

One need not blindly test all those possibilities.

First, note each factor has form 6m+73 = 1 (mod 3).
This excludes factorizations into negative factors,
since they are all = −1 (mod 3).

Second, using the symmetry n −> −n we may assume
n is positive and hence that 6 (k+n) + 73 is the
largest factor b in 5161 = a b. By elimination

n = (b−a)/12, k = n + (a−73)/6

This leaves only a,b = 13,397; 1,5161

with solutions n,k = 32, 22; 430,418

−−Bill Dubuque
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- ***References***:
  - ***help with diophantine equation***
    - *From:* mechmech
  - ***Re: help with diophantine equation***
    - *From:* john_ramsden


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