Re: the concept of a representation of a group

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- *Date*: Thu, 25 Aug 2005 17:07:50 +0300

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m1ngleg02@xxxxxxxxxxxxxx wrote:

> Thank you for your detailed answer Igor. About the dimension
> of the
> group, I restrict to the case of Lie groups which can be
> represented by
> matrix groups.

You should have immediately told us that you are only interested in the representations of Lie groups. Igor Khavkine's reply (and my first impression also) was based on the belief that you were asking about the representation theory of finite groups (which is what people IMHO mean, when they talk about representation of groups without specifying the type of group). His good points (a) and
(b) in particular only apply to finite groups. Ok, of course there are a lot of similarities, but the most marked difference is that for finite groups you can give a complete
finite list of irreducible reps − for Lie groups you can
only describe the irreducible reps in terms of certain
parameters (like highest weights etc.)

With the dimension of the group I mean the number of

> parameters of the matrix group. What I do not understand is exactly the
> relation between the dimension of the group and the dimension of their
> representations.

Well that depends. A representation is only a homomorphism of groups. It doesn't respect linear structure: e.g. should it happen that a linear combination C=a*A+b*B of two matrices A and B in, say SU(n), is again a matrix from that
same group (for some choice of scalars a and b), it doesn't
follow that

rho(C)=a*rho(A)+ b*rho(B)

(here rho is the homomorphism). Consequently the linear span of the image may be a lot higher. Furthermore, you get a lot more linear mappings by composing linear combinations of all the matrices in the image of rho etc.

You may benefit from the study of the universal enveloping algebra of the Lie algebra of your group. This are actual algebras (= combinations of vector spaces and rings), and there you could count dimensions in the linear algebra style: dimension of the image= dimension of the range − dimension of the kernel etc. Alas, the universal enveloping algebra, of even a small group like SU(2) is infinite dimensional. This explains why there is no upper bound to the dimension of an irreducible rep of SU(2).

> You may mean normal commuting matrices in a), when talking about simultaneously diagonalizable matrices. You maybe mean in the last sentence of b), that with help of the natural representation (forgetting that I do not know what it is exactly) one can find the fundamental representations, which build up the irreducible representations by tensor products. I looked regular representation up in the internet, but I found for SU(n), the regular representation is the adjoint representation. Is the regular representation the adjoint representation?

The two concepts are rather different. Both do serve a similar purpose as a starting point for finding other reps.

> After all my question remains unanswered. Is the dimension of a representation the dimension of H? Can the quotient group G/ker(f) be used for something? Maybe you could explain it with the help of a small simple group, SU(2).

Look at texts explaining the relation of reps of a Lie group and reps of the universal enveloping algebra of its Lie algebra.

Cheers,

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