

Re: Cardinality of Real Numbers

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- *From:* David C. Ullrich <ullrich@xxxxxxxxxxxxxxxxxxxx>
 - *Date:* Sun, 28 Aug 2005 14:03:18 -0500
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On 28 Aug 2005 10:54:01 -0700, jswimr3@xxxxxxxx wrote:

>I've been thinking about cardinality proofs lately, and I've run into
>something that's been bothering me. I thought of what seems like a
>mapping from the set of integers to the set of real numbers. Now, of
>course, this can't exist, so there must be something wrong with my
>mapping, but I can't see what it is.

First, terminology: There certainly do exist mappings from the integers to the set of reals, for example if you map each integer n to n that defines such a mapping. What does not exist is a mapping from the integers onto the set of reals – that is, a mapping such that every real is the image of some integer under the mapping.

Now for the details:

>The mapping works like this: for each integer, map it onto all the
>reals you can get by putting a decimal point anywhere in it. For
>example, 123 would map to:

>
>123
>12.3
>1.23
>.123
>

>It seems like this would cover the full set of real numbers.

It's already been pointed out that this does not cover every real, in fact nothing here gets mapped to $1/3$.

Much worse, this is not a "mapping" at all! Because you "map" a given integer to more than one real – that's not allowed by the definition of "map".

(If we are allowed to map integers to more than one real then there's no problem covering all the reals, simply map 0 to every real.)

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>Each of
>these mapped sets of reals is finite, and there would be a countable
>number of these sets, since the integers are countable. So this would
>seem to be a countable union of finite sets, which would, itself, be
>countable.
>
>I was wondering if perhaps I run into trouble with real numbers like
>.00000123, which wouldn't correspond to an integer in my scheme. But
>it seems like you could get around that by making a new rule, for
>example, that real numbers which begin with 1 would map to the numbers
>they would normally map to, but would also map to decimals where the 1
>is turned into a zero. So 10000123 would map to all the numbers it
>normally maps to, and would also map to .00000123. There would still
>be a finite number of real numbers for each integer.
>
>But the real numbers aren't countable. So where did I go wrong?
>
>Thanks,
>John

David C. Ullrich

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• **References:**

- ◆ **Cardinality of Real Numbers**
 - ◇ From: jswimr3
- Prev by Date: **Re: 0.999... = 1? (I know, a beaten dead horse)**
- Next by Date: **Re: infinity**
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