Cardinality of Real Numbers

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I've been thinking about cardinality proofs lately, and I've run into something that's been bothering me. I thought of what seems like a mapping from the set of integers to the set of real numbers. Now, of course, this can't exist, so there must be something wrong with my mapping, but I can't see what it is.

The mapping works like this: for each integer, map it onto all the reals you can get by putting a decimal point anywhere in it. For example, 123 would map to:

123
12.3
1.23
..123

It seems like this would cover the full set of real numbers. Each of these mapped sets of reals is finite, and there would be a countable number of these sets, since the integers are countable. So this would seem to be a countable union of finite sets, which would, itself, be countable.

I was wondering if perhaps I run into trouble with real numbers like ..00000123, which wouldn't correspond to an integer in my scheme. But it seems like you could get around that by making a new rule, for example, that real numbers which begin with 1 would map to the numbers they would normally map to, but would also map to decimals where the 1 is turned into a zero. So 10000123 would map to all the numbers it normally maps to, and would also map to .00000123. There would still be a finite number of real numbers for each integer.

But the real numbers aren't countable. So where did I go wrong?

Thanks,
John

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