Re: change of variable in multiple integrations...

Source: http://sci.tech−archive.net/Archive/sci.math/2005−09/msg00035.html

- *From*: "Proginoskes" <CCHeckman@xxxxxxxxx>

- *Date*: 31 Aug 2005 13:59:30 −0700

Proginoskes wrote:
> kiki wrote:
> > "Proginoskes" <proginoskes@xxxxxxxxxxxxx> wrote in message
> > news:1123058829.408325.6100@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
> > >
> > > kiki wrote:
> > >> Hi all,
> > >>
> > >> I understand the change of variables in bi−varite integration:
> > >>
> > >> For example, if you have a<x1<x2<b, and you want to do integration:
> > >>
> > >> Integrate(Integrate(f(x1, x2), w.r.t x2 from x1 to b), w.r.t x1 from a to
> > >> b)
> > >>
> > >> If I change order of integration, it becomes
> > >>
> > >> Integrate(Integrate(f(x1, x2), w.r.t x1 from a to x2), w.r.t x2 from a to
> > >> b)
> > >>
> > >> This I understand well.
> > >>
> > >> How about more variables: for example, a<x1<x2<x3<x4<b,
> > >>
> > >> Original ordering: (first of all, is this a correct one)?
> > >>
> > >> Integrate(Integrate(Integrate(Integrate(f(x1, x2, x3, x4), w.r.t x4 from
> > >> x3
> > >> to b), w.r.t x3 from x2 to b, w.r.t x2 from x1 to b, w.r.t x1 from a to
> > >> b)
> > >>
> > >> Then change the order of x2 and x4: (is the following correct?)
> > >>
> > >> Integrate(Integrate(Integrate(Integrate(f(x1, x2, x3, x4), w.r.t x2 from
> > >> x1
> > >> to x3), w.r.t x3 from x1 to x4, w.r.t x4 from x1 to b, w.r.t x1 from a to
> > >> b)
> > >

> > > Yes, but I think your answer will be off by a sign. When you change
> > > variables, you need to multiply f(...) by the determinant of a certain
> > > matrix. This is why when you do integration with polar coordinates,
> > > you have the extra r factor. In short:
> > >
> > > Change from (x,y) to (r,T) [T for theta]:
> > > x = r cos T
> > > y = r sin T
> > >
> > > dx/dr = cos T, dx/dT = −r sin T
> > > dy/dr = sin T, dy/dT = r cos T
> > >
> > > | cos T −r sin T |
> > > | sin T r cos T |
> > >
> > > = r cos^T − (−r) sin^T = r (cos^T + sin^T) = r.
> > >
> > > Thus: int (f(x,y) dy dx) = int(f(r cos T, r sin T) r dr dT).
> > >
> > >> As you can see, these kind things can be very confusing when number of
> > >> variables become larger... is there any general rules governing such
> > >> techniques?
> > >
> > > Yes. Look up "Jacobian" in a 3−semester calculus textbook, or
> > > http://mathworld.wolfram.com/Jacobian.html .
> > >
> > > −−− Christopher Heckman
> > >
> > > P.S. This seems correct; if anyone sees any obvious mistakes, please
> > > let me know, because I'll be teaching this in a month or so and haven't
> > > touched Calc III since my undergraduate days ...
> > >
> > > P.P.S. No, it was my senior year in high school, Fall 1988.
> > >
> >
> > Hi Chris,
> >
> > I understood the Jacobian and change of polar coordinates etc. But I still
> > did not see where did I miss a "sign"?
> >
> > Could you please elaborate directly on this problem...
>
> You're swapping variables x2 and x4. This is like using the following
> transformation:
>
> y1 = x1
> y2 = x4
> y3 = x3
> y4 = x2
>
> Now you need to calculate partial derivatives. The Jacobian becomes

>
> | 1 0 0 0 |
> | 0 0 0 1 |
> | 0 0 1 0 |
> | 0 1 0 0 |
>
> The determinant of this matrix is −1, so the new integral is of
> f(x1,x4,x3,x2) times −1, not just f(x1,x4,x3,x2).

Nope. (No one caught this.) The Jacobian is the absolute value of the determinant, so you're still integrating the same function.

−−− Christopher Heckman

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