Re: Cardinality of Real Numbers

Source: http://sci.tech−archive.net/Archive/sci.math/2005−09/msg00326.html

- *From*: Martin Shobe <mshobe@xxxxxxxxxxxxx>
- *Date*: Fri, 02 Sep 2005 04:36:46 GMT

On 31 Aug 2005 21:11:37 −0700, "Ross A. Finlayson"
<raf@xxxxxxxxxxxxxxx> wrote:

>Martin Shobe wrote:
>> On 31 Aug 2005 18:10:12 −0700, "Ross A. Finlayson"
>> <raf@xxxxxxxxxxxxxxx> wrote:
>>
>> >Jonathan Hoyle wrote:
>> >> I think the issue is that without the Axiom of Choice, you cannot
>> >> assume that all cardinals are orderable. That is to say, assuming the
>> >> Axiom of Choice and given any two arbitrary sets S1 and S2, we know
>> >> that exactly one of the following three holds:
>> >>
>> >> 1. |S1| = |S2| (there is a bijection between S1 and a S2)
>> >> 2. |S1| < |S2| (there is a bijection between S1 and a proper subset of
>> >> S2, but not between S1 & S2)
>> >> 3. |S1| > |S2| (there is a bijection between a proper subset of S1 and
>> >> S2, but not between S1 & S2)
>> >>
>> >> Without AC, you cannot assume that exactly one of the above three must
>> >> hold. You could have a case in which S1 and S2 are incomparable; that
>> >> is to say, none of the above conditions hold.
>> >>
>> >> The wikipedia entry may get you started:
>> >> http://en.wikipedia.org/wiki/Cardinal_number
>> >>
>> >> As for the reals, the Axiom of Choice proves that there exists a
>> >> well−ordering of them.
>> >>
>> >> Good Luck!
>> >>
>> >> Jonathan Hoyle
>> >
>> >There is a simple existence proof of an ordinal that bijects to that
>> >set S1, correct?
>>
>> Not without using the axiom of choice.
>>
>> >Where that is so, there is guaranteed the existence of an ordinal

>> >bijecting to any set, so any set is well−orderable.
>> >
>> >That is to say, for S1, regardless of what S2 is, there exists at least
>> >one ordinal that bijects with S1: true?
>>
>> Not necessarily. There are models of ZF where this isn't true.
>>
>> >If a set has a cardinality, then there is an equivalent ordinal, and
>> >the set is well−orderable. From what I understand of you saying so,
>> >some sets have varying cardinality?
>>
>> No. They don't have any, um, ordinality.
>>
>> > Doesn't that violate a trichotomy
>> >of cardinals, and thus infinite sets are equivalent?
>>
>> ZF can't prove the trichotomy of cardinals (that requires the axiom of
>> choice). But regardless of whether it can or can't, it can still show
>> that bijections form an equivalence relation, and that there is more
>> than one equivalence class of infinite sets.
>>
>> >So anyways, besides the discussion about ramifications of well−ordering
>> >the reals, which are a well−orderable set in ZFC for everybody, does
>> >the existence of an ordinal for each cardinal, even if it is not simple
>> >to say what it is, does the existence of said ordinal imply a
>> >well−ordering exists for that set, any set? Similarly, there is the
>> >existence proof of a well−ordering of the reals in ZFC, and if the
>> >reals are a set, then they are well−orderable, and the well−ordering
>> >implies the extension of Cantor's first to that there must be adjacent
>> >points, regardless of whether it implies that there are more than
>> >countably many disjoint intervals.
>>
>> NO. In order to reach the contradiction, Cantor's first requires that
>> the well−ordering be order−equivalent to the natural numbers. Since
>> any well−ordering of the reals will not be order−equivalent to the
>> natural numbers, no contradiction is reached, and there are no
>> adjacent points or uncountable collections of disjoint intervals.
>>
>> Martin
>
>Heh heh heh heh.
>
>I'm talking about an extension of Cantor's first, where for any
>well−ordered set that bijects to the reals, there are that many
>disjoint intervals generated.

You have yet to demonstrate that this is possible.

>What do you mean: "Cantor's first requires the well−ordering be
>order−equivalent to N?" Do you mean that to say that Cantor's first
>applies to a bijection from N to R only, or what?

Yes. Cantor's first assumes the existance of a bijection between the natural numbers and the reals. From this, a contradiction is reached by showing that there must be a real mapped to a natural number that is also mapped to a number larger than any natural number.

> I'm talking about
>any set mapping to R, in extension. What do you mean by
>"order−equivalent"?

Order equvalent means that there exists a bijection between two sets such that the order is preserved.

>With regularity in ZF, is the existence if not identity of an ordinal
>for each cardinal certain?

No.

Martin
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- ***References***:
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    - *From:* Ross A. Finlayson
  - ***Re: Cardinality of Real Numbers***
    - *From:* Martin Shobe
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