

Re: Cardinality of Real Numbers

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- *From:* "Randy Poe" <poespam-trap@xxxxxxxxxx>
 - *Date:* 2 Sep 2005 06:29:05 -0700
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Martin Shobe wrote:

> On Thu, 01 Sep 2005 23:39:16 -0600, Virgil
> <ITSnetNOTcom#virgil@xxxxxxxxxxxx> wrote:
>
>> In article <u7lfh19ci265649u3nurks2u3b1281csc6@xxxxxxxx>,
>> Martin Shobe <mshobe@xxxxxxxxxxxx> wrote:
>>
>>> >What do you mean: "Cantor's first requires the well-ordering be
>>> >order-equivalent to \mathbb{N} ?" Do you mean that to say that Cantor's first
>>> >applies to a bijection from \mathbb{N} to \mathbb{R} only, or what?
>>>
>>> Yes. Cantor's first assumes the existence of a bijection between the
>>> natural numbers and the reals. From this, a contradiction is reached
>>> by showing that there must be a real mapped to a natural number that
>>> is also mapped to a number larger than any natural number.
>>
>>As I read it, Cantor's first starts with an arbitrary injection from the
>>naturals to the reals, and shows that there is some real not in the
>>image of that injection. Thus no such injection can be a surjection. No
>>contradiction required.
>>
>>Many mathematicians, and I believe Cantor was one of them, did not much
>>like proofs by contradiction, and go to considerable lengths to avoid
>>them where possible. In this case no great lengths were required.
>
> This is where I got Cantor's first proof from
>
> http://en.wikipedia.org/wiki/Cantor%27s_first_uncountability_proof
>
> In this article, the proof is a proof by contradiction. As I don't
> have access to the originals, I can't tell you if that was actually
> Cantor's proof.

As with the diagonal proof, the proof sketched there does not require the contradiction form. There is no reason to assume, as the first line states, "some sequence x_1, x_2, x_3, \dots has all of \mathbb{R} as its range."

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Replace that with: "Let x_1, x_2, x_3, \dots be a sequence of reals".

The proof shows that there exists c not in the sequence. Thus the original sequence, which had no restrictions put on it, is incomplete. Therefore there does not exist a sequence which includes all of the reals.

– Randy

• *References:*

- ◆ ***Re: Cardinality of Real Numbers***
 ◇ *From: Ross A. Finlayson*
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-
- Prev by Date: ***Re: The Measure of Beauty Created in Nature***
 - Next by Date: ***Re: Geometry question***
 - Previous by thread: ***Re: Cardinality of Real Numbers***
 - Next by thread: ***Re: Cardinality of Real Numbers***
 - Index(es):
 - ◆ ***Date***
 - ◆ ***Thread***