Re: sin x / x tends to 1...

Source: http://sci.tech−archive.net/Archive/sci.math/2005−09/msg00703.html

- *From*: lrudolph@xxxxxxxxx (Lee Rudolph)
- *Date*: 3 Sep 2005 17:16:18 −0400

David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx> writes:

>On Sat, 3 Sep 2005 19:01:29 +0000 (UTC), Darren J Wilkinson
><d.j.wilkinson@xxxxxxxxx> wrote:
....
>>Is there an easy way to show that
>>aradians and radians are the same?
>
>In the sense in which I would use the word "show",
>I doubt it − I don't think that we can even give an
>easy definition of area, much less arc length.

We can certainly give an "easy" (also correct) definition of "area" for polygonal regions, which becomes all the easier if we restrict to convex polygonal regions (since then the existence of a dissection into interior−disjoint triangular regions is *completely* obvious). To show that this definition *is* a definition (i.e., is independent of choices) is also elementary, if somewhat tedious, and is in Euclid. Then we can at least say with some hope of being convincing to that bright 15−year old that this definition of area extends consistently to a larger class of regions, in particular, regions (like that bounded by the circle) which can be squeezed between polygonal regions the difference of whose areas (already defined) can be made arbitrarily small. The curtailed theory of area we can develop like that is certainly enough to make your proof with aradians fully justifiable.

Now, as you suggest, a similarly elementary theory of arc length for non−polygonal curves is lacking. But all we *need* is a theory of arc length for arcs of circles! And there, our bright 15−year old will be charmed by the definition of length that comes out of the usual proof that the derivative of the area of a circle with respect to the radius is the circumference by reversing the roles of some definitions and some conclusions. (Of course the proof also works for circular sectors.) I call that quite adequately

elementary, for appropriate values of "bright".

Doing this for yer average freshstudent in the USofA
would be a fish of a different kettle, unfortunately.

Lee Rudolph

>But in the sense in which I "showed" something in
>my previous post then yes, I believe there is an
>easy way, although I don't see exactly how to
>do it (which is why I decided to give an argument
>based on area this morning).
>
>Given our definition of aradian, the definition
>we gave for pi amounts to saying that pi is
>the area of the unit circle, and showing that
>aradians are the same as radians then amounts
>to showing that the circumference is 2pi. So
>we have to show that the circumference is twice
>the area. (And showing that amounts to the
>same thing as giving a proof that sin(x)/x
>tends to 1 based on arclength and radians
>instead of area...)
>
>You might note that if we're willing to
>believe a little bit then it's _very_ easy:
>Say P_n is a regular n−gon inscribed in the
>unit circle. It's easy to show that the ratio
>length(P_n)/area(P_n) tends to 2 as n tends
>to infinity, because that ratio is exactly
>the length of the segment from the center to
>the midpoint of one of the sides of P_n.
>So if we're willing to just believe that
>the length of P_n tends to the length of
>the circle and the area tends to the area
>we're done. That might count as "showing"
>it for your purposes − it bothers me, because
>of examples where this curve approaches that
>curve although the length does not approach
>the length.
>
>Seems like we should be able to do better,
>giving actual inequalities as this morning,
>but I don't see exactly how to do it.
>The point:
>
>Say Q_n is a regular n−gon _circumscribed_ about
>the unit circle. If we could show that the length
>of Q_n was greater than the circumference of the
>unit circle we'd be set; the fact that a straight

>line is the shortest distance between two points
>shows that the length of P_n is less than the
>circumference and we have the inequalities
>we want. But the the fact that the length of
>Q_n is greater than the circumference of the
>unit circle is what I didn't see how to prove
>this morning on the basis of anything plausible
>as plausible as "a straight line is the shortest
>distance between two points), although it certainly
>looks like it's true (as of course it is). If you
>can prove this inequality you're set.
>
>Or for that matter if you can prove that
>the length of Q_n is larger than some c_n
>times the circumference of the unit circle,
>where c_n −> 1, you're set. I thought I
>had a proof of that a second ago, based on
>using a larger circle in which Q_n is
>inscribed, but the inequality seems to go
>the wrong way.
>
>>Regards,
>
>
>**********
>
>David C. Ullrich
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- ***Follow−Ups***:
  - ♦ ***Re: sin x / x tends to 1...***
    - ◊ *From:* David C . Ullrich
  - ♦ ***Re: sin x / x tends to 1...***
    - ◊ *From:* Darren J Wilkinson

- ***References***:
  - ♦ ***sin x / x tends to 1...***
    - ◊ *From:* Darren J Wilkinson
  - ♦ ***Re: sin x / x tends to 1...***
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- Prev by Date: ***Re: A conjecture about maximal planar graphs***
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