derivatives as limit of a sequence of continuous functions

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I'm trying to prove the following fact: if f:I −−> R is differentiable on an open interval I, then f' is the limit of a sequence of continuous functions defined on I.

First, I considered an interval like (a, oo) with a real or −oo. Let t_n be a sequence of positive numbers that converges to 0 and, for every positive integer n, define g_n(x) = (f(x + t_n) − f(x))/(t_n). Then, it's easy to see that each g_n is continuous and g_n −> f'. For intervals like (−oo, a) the proof is similar.

A trickier case happens if we have intervals like (a,b), with a and b real. Admitting f has a limit L at b, we can suppose t_n is in (0, b−a) for every n and define g_n(x) = (f(x + t_n) − f(x))/(t_n) if a < x < b − t_n and g_n(x) = (L − f(b −t_n))/(t_n) if b−t_n <= x < b. Then each g_n is continuous on (a, b) and, for sufficient large n, every x in n (a,b) will satisfy x < b − t_n, so that g_n => f'.

But I couldn't handle the case (a,b), and b real, if f doesn't have a limit at a nor at b. In this case, the previous reasonings donn't work and I'd like a suggestion.

Thank you
Artur

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