Re: sin x / x tends to 1...

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- *From*: Darren J Wilkinson <d.j.wilkinson@xxxxxxxxx>
- *Date*: Tue, 6 Sep 2005 08:12:42 +0000 (UTC)

Ronald Bruck <bruck@xxxxxxxxxxx> wrote:
> In article <dfi87o$gl1$1@xxxxxxxxxxxxxxxxx>, Darren J Wilkinson
> > Now let's look at circumference. We can look at the perimeter of the
> > inscribed N−gon, and see what happens as N increases. There isn't much
> > point looking at the perimeter of the circumscribed polygon, because it
> > isn't "obvious" that its perimeter is greater than the circumference of
> > the circle.
>
> Sure it is. Let L_n be the perimeter of the inscribed polygon, M_n the
> perimeter of the circumscribed polygon (both regular, of n sides). A
> little work shows L_n is increasing and M_n is decreasing (but we
> really only need this for powers−of−2, which is still less work).
> Manifestly L_n < M_n, because the circumscribed polygon is a scalar
> multiple > 1 of the inscribed polygon, rotated slightly−−which clearly
> doesn't affect the perimeter. We conclude that L_n < M_m for any m and
> n (because L_n < L_{mn} < M_{mn} < M_m).
>
> If the perimeter of the circle is defined as the sup of the L_n, it's
> now clear that M_n is > the perimeter.

Doh! That is _assuming_ the (modern) definition of arc length. I'm
afraid the rather disappointing conclusion I'm coming to is that the
bright 15 year old has a better understanding of the "true nature" of
length and area than most professional mathematicians! ;−) Now lest this
thread finishes with the impression that I don't _believe_ the limit,
let me set your minds at rest. I _know_ the limit is true, because it is
"obvious" (to me) that the circle has the smallest perimeter of any
shape with a given area. Now since the circumscribed polygon clearly has
greater area than the circle, it must have longer perimeter (without any
recourse to a modern definition of arc length). Then, we
can squeeze the perimiter in the usual way, and we conclude that the
limit is true, that a circle has area pi, that the polygonal sup
definition of arc length makes sense, and that that whole of mathematics
is saved! Hurrah! Hurray!

Now the only slight problem with the above argument is that whilst it is
"obvious" that the circle has the smallest perimeter of any shape with a
given area (cue analogies with spherical soap bubbles and elastic bands
around bundles of dried spaghetti), it doesn't seem particularly

elementary (so, I bet that given modern definitions of length and area,
it is a pain in the butt to prove). I was just hoping that there would
be a more elementary way to see the limit. If there isn't, that's a bit
of a shame, but that's just the way it is. But then I wish you'd just
say so (as several people have!), instead of pretending that I'm stupid
for even asking the question, because it is "obvious" from the (modern)
definition (which _makes_ it obvious).

Au revoir,

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