

Re: what makes it true?

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- *From:* grubb@xxxxxxxxxxxxxxxxxxxx (Daniel Grubb)
 - *Date:* 8 Sep 2005 19:46:33 GMT
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>> I can understand, given a formal system, whether I have a well formed
>> formula, a proof, etc because I can apply whatever rules are given
>> for forming formulae or for deriving new strings from the ones already
>> obtained. However, without a set theory, I can't talk about the set of
>> statements or prove anything about proofs. So my understanding of
>> formal systems would be purely local if not for some type of set theory.

> All of this sounds very strange. I don't see how it relates to
> anything that actually happens. To begin with, what do you mean by
> being given a formal system? In the real world, this means being
> given such explanations as " $A \rightarrow (B \rightarrow A)$ is an axiom for all formulas
> A and B ". We understand such explanations perfectly well without
> any set theory whatsoever, and I can't imagine by which procedure
> one would first introduce a set theory and then somehow use that
> set theory to elucidate the explanation.

Like I said, I can follow the rules of the system without having a set theory. However, to prove anything about the system, such as consistency, I have to be able to talk about the set of statements in the theory, so I need a set theory. To be able to talk about the set of provable statements, I'll need to actually talk about sets. To talk about truth in the theory, I need to talk about models of the theory. Since a model is a set, that requires a set theory. To ask whether a statement is independent from the theory, I need to be able to talk about whether it is in the set of provable statements from the theory. I don't see any way around it.

—Dan Grubb

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- *Follow-Ups:*
 - ◆ *Re: what makes it true?*
 - ◇ *From:* Torkel Franzen

- *References:*
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