Re: infinity

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>> Fraenkel didn't necessarily think so. Do you think Fraenkel
>> was a crank? I ask because you seem so knowledgeable
>> of them.

No, I do not.

>> Anyways I've heard of some independence result of Choice
>> before and what is it. I've never actually verified that. I hope
>> you could outline the independence result of choice from ZF,
>> particularly in light of where for each cardinal, there is a
>> corresponding well−ordered ordinal, and even the thread "The
>> Sufficiency of the Axiom of Choice", from some years ago.

Yes, excellent memory, as that was a number of years ago. I don't know the proof well enough to outline it for you, but it is a rather famous proof. The proof that AC is consistent with ZF was written by Kurt Godel in 1940. In 1963, Cohen completed the proof by proving the full independence of AC to ZF. That is to say, if either ZFC or ZF + ~AC were inconsistent, then it is only because ZF itself is inconsistent.
You might consider finding the Springer−Verlag book, "Zermelo's Axiom of Choice: Its Origins, Development, and Influence" by Gregory Moore (1982, ISBN: 0387906703).

>> The set of all ordinals is an ordinal. You say "oh, it's a
>> proper class." Well, there can only be one proper class,
>> besides that in a theory where sets are the objects
>> proper classes are outside the entire domain of
>> discourse, which doesn't exist.

Not necessarily. There are minimal extensions to ZFC which do deal with proper classes, such as von Neumann−Bernays−Godel Set Theory. In this framework, the class of ordinals, the class of cardinals, etc. all exist. However, you are correct that if we stay restricted within the specific confines of ZFC (or just ZF for that matter), "the set of all ordinals" does not exist, and therefore is not an ordinal. This is easily proven using the Axiom of Foundation.

>> What's the class of all classes?

This class is outside even the range of von Neumann−Bernays−Godel Set Theory. The Springer−Verlag book, "The Joy of Sets" by Keith Devlin goes into non−Foundation Set Theories and does describe this "universal" class. Directed graphs are used for modelling it, and this class (symbolized by the capital omega) is depicted as a single vertex with a directed edge pointing to itself.

>> There can't be one? Then, there can only be one
>> proper class. (It's the ur−element of the null axiom
>> theory and a set.)

I'm not sure how you made that particular leap. However you did it, it is untrue. Neither ZFC nor NBG have a "class of all classes", but ZFC has no proper classes whereas NBG has many.

>> It was Cantor who promoted his domain principle in the
>> era of naive set theory that the universal set, or set of all
>> sets, exists. Is Cantor a crank?

No, nor was Newton a crank just because he didn't believe in relativity. Cantor's naive set theory preexisted to ZFC, and was in fact the inspiration fror Zermelo and Fraenkel to develop it.

>> No, ZF is inconsistent, and infinite sets are equivalent.
>> I'm not talking about typos.

You have a proof of the inconsistency of ZF? That I find rather unlikely. More likely is that the results of ZF contradict some pre−existing notions in your mind, and does not in fact contradict anything in mathematics. This does not mean ZF is inconsistent; it merely means that ZF is inconsistent with Ross Finlayson Set Theory (which is fine, as Ross Finlayson Set Theory is probably inconsistent on its own).

>> Now, "demonstrably correct results" is something that is
>> interesting. I'm interested in learning more about that.

Okay, it is an example from probability (one I gave to Tony in a previous post). Consider a simple game with players A and B each flipping a fair coin in turn. The first one flipping heads wins, and Player A goes first. What is the probability that A will win?

Well, A will flip heads and win on the first flip with probability 1/2. A can also win by flipping tails on the 1st flip, B flipping tails on the 2nd, and A flipping heads on the 3rd, with probability 1/2 x 1/2 x 1/2 = 1/8. Etc. etc. So the total probability that A will win is the infinite sum:

$1/2 + 1/8 + 1/32 + ... = 2/3$

We can perform this sum only thanks to Countable Additivity. Without it, you cannot solve this simple problem. Yet the correct answer is indeed 2/3 as is demonstrated by writing a simple computer program to run through a very large number of random games. Claim all you want that "an infinite sum makes no sense", but the probability of this outcome is demonstrable and agrees with our countable sum.

Uncountable Additivity however does NOT work, as we see in Measure Theory (Probability Theory's brother): the measure of the interval [0,1] is 1, but each individual point in [0,1] is of measure 0, and assuming Uncountable Additivity would sum 0 + 0 + ... to 0, not 1. So in a very real sense, which infinite size the set is determines what you can do with it.

>> Well−order the reals, I dare you.

Okay, take any arbitrary 1−1 mapping F between the reals R and the power set of natural numbers P(N). By the Axiom of Choice, we know we can well−order P(N), so take any such well−ordering, <=. Define a <= operation (obviously different from the standard one) on R such that for a,b in R, a <= b whenever F(a) <= F(b). You have now well−ordered R. Creating F and well ordering P(N) are left as exercises. :−)

Hope that helps,

Jonathan Hoyle

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