Re: Cantor

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Pedro wrote:
> I think I'm getting sidetracked by giving poor examples. Can't see the
> forest for the trees, and all that... Let me ask it one more way, if I may.
>
> It's obviously impossible to actually present a "complete" list of reals
> prior to initiating the diagonalization method as the list is infinitely
> long. Rather, the diagonalization method is a thought experiment that says,
> in effect, "no matter how complete one makes the list, one can use this
> typographical method to find with certainty a number not on the list".
>
> But I can make the same conceptual argument about the natural numbers (or
> non−negative integers, if you prefer). In my example below, SSSS0 differs in
> the first position from 0, differs in the second position from S0, etc...,
> thereby giving a number that differs from all others on the list and
> demonstrating an absurdity: that the natural numbers are not enumerable.
> (The only new assumption / constraint I've placed on this example is that
> the list be ordered. That doesn't seem to be an unreasonable thing to do
> with the natural numbers.)
>
> So I'm obviously missing something fundamental about the whole
> diagonalization thing. Any idea what it is?

This point is made below by James Dolan. I'm just putting in
my two cents worth.

In the "diagonalization" proof that [0,1] is uncountable, the
form of the proof is:

1. Let f:N−>[0,1] be any mapping from N to [0,1].
2. There exists x in [0,1] such that x does not equal
f(n) for any n in N.

The construction of x in step 2 relies on a theorem that
any string of digits 0.ssssss... represents an x in [0,1].

Remember, the purpose of the construction is not to produce
a sequence, but to generate a single value x which, by the
construction, is not mapped by any f(n). This number x has

the property that its n−th digit differs from the n−th digit of f(n).

So the corresponding proof in the natural numbers should be trying to construct a corresponding m in N. But you do not have the corresponding theorem that any (possibly infinite) sequence of digits represents a natural number. Therefore the construction can not be guaranteed to produce such a number.

− Randy

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