

Re: " Wanted the r iterated of $f(x) = 2*x/(1-x^2)$, $f^{[r]}(x)$ "

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Source: <http://sci.tech-archive.net/Archive/sci.math/2005-10/msg00655.html>

- *From:* rusin@xxxxxxxxxxxxxxxxxxxxxxxx (Dave Rusin)
 - *Date:* 7 Oct 2005 14:26:40 GMT
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In article <1128665817.093842.197060@xx>, <alainverghote@xxxxxxxx> wrote:

>Do some of you know a closed form
>for the rth iterate of $2*x/(1-x^2)$.

.... which equals $\tan(2u)$ if $x = \tan(u)$. That is, if $f(x) = 2x/(1-x^2)$ then $f(\tan(u)) = \tan(2u)$, so that $f(f(\tan(u))) = \tan(4u)$ and $f(f(f(\tan(u)))) = \tan(8u)$, etc. In general you can then prove by induction that $f^r(\tan(u)) = \tan(2^r u)$ for positive integers r .

You can write this as $f^r(x) = \tan(2^r \arctan(x))$ I suppose but this is only because, for a given x , the possible values of u all differ by integer multiples of π and thus the possible values of $2^r \arctan(x)$ will also differ by integer multiples of π , giving them all the same values in $\tan()$. I say this just to emphasize, as always must be done, that this analysis is predicated on the assumption that r is an integer; there is nothing in the definition of f alone that tells us what f^r even *means* when r is not an integer.

dave

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- *Follow-Ups:*
 - ◆ *Re: " Wanted the r iterated of $f(x) = 2*x/(1-x^2)$, $f^{[r]}(x)$ "*
◇ *From:* alainverghote
 - *References:*
 - ◆ *" Wanted the r iterated of $f(x) = 2*x/(1-x^2)$, $f^{[r]}(x)$ "*

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◇ From: alainverghote

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