

Re: Fourier transform and the like

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- *From:* israel@xxxxxxxxxxx (Robert Israel)
 - *Date:* 11 Oct 2005 23:18:45 GMT
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In article <1129058622.804690.94460@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>, S. Gammelmark <gammelmark@xxxxxxxx> wrote:
>I'm well aware, that e^{ikx} does not form an orthonormal/orthogonal
>basis with respect to the same inner product as is used in the fourier
>transform/series, but it can, however, be constructed using the
>Gram-Schmidt process.

I'm not sure what "it" is here, but if you're talking about the real line here in the context of Fourier transform, e^{ikx} is not square-integrable, so Gram-Schmidt can't even get started.

Another way of thinking of Fourier transform is as the application of the Spectral Theorem to the self-adjoint operator $-i d/dx$ on $L^2(\mathbb{R})$. This operator is rather special because it generates translations. Other self-adjoint operators with continuous spectrum would produce other transforms. Self-adjoint operators with discrete spectrum lead to orthonormal bases, analogous to Fourier series. If you want a nice orthogonal basis for $L^2(\mathbb{R})$, you might try Hermite functions $u_n(x) = \pi^{-1/4} 2^{n/2} (n!)^{-1/2} \exp(-x^2/2) H_n(x)$ (H_n the Hermite polynomials), which are eigenfunctions for the harmonic oscillator.

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