Re: circle homeomorphism

Source: http://sci.tech−archive.net/Archive/sci.math/2005−10/msg01279.html

- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx>
- *Date*: Thu, 13 Oct 2005 06:00:57 −0500

On 12 Oct 2005 15:54:33 −0400, lrudolph@xxxxxxxxx (Lee Rudolph) wrote:

>David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx> writes:
>
>>On Tue, 11 Oct 2005 21:23:04 +0100, "Dumdum project"
>><yeah@xxxxxxxxxxx> wrote:
>>
>>>Im just trying to show:
>>>
>>>p(f^n)=n.p(f) where p is the rotation number, and f is any circle
>>>homeomorphism, im not really sure where to start. i know its quite
>>>elementary, and have convinced myself by drawing pictures.
>>
>>What is the "rotation number" of a homeomorphism of the circle?
>>
>>(If I had to guess what it was, it would always be 1 or −1, which
>>seems like it can't be what you mean, given your question...
>>
>>Unless the rotation number is what I'd guess it was and you
>>really meant to ask about proving that p(f^n)=p(f)^n?)
>
>I can never remember the definition of "rotation number"
>but it's classic stuff, going back at least to Birkhoff (pere)
>and Poincare. What I *think* I remember about it, which is
>certainly not the definition, is that not only does Homeo^+(S^1)
>(the group of orientation−preserving homeomorphisms of S^1,
>equipped with some natural topology, presumable the compact−open
>topology) deformation−retract onto its subgroup SO(2), but there
>is a particularly natural deformation−retraction r, and the
>rotation number of a homeomorphism h is the fraction of 2\pi
>through which r(h) is an honest rotation.

Huh. When you see a word you don't know around here you can
never tell whether it's really a word you don't know or a
garbled version of a word you know...

>.... Oh, heck, I'll
>use Google. ... It's all here:
>

>http://hopf.math.northwestern.edu/AMS_address.pdf

Huh. Hint for the OP:

It seems we have an increasing homeomorphism F of R,
with period 2\pi, such that

f(eit(t)) = eit(F(t))

(writing eit(t) = exp(2 Pi i t)); F is a "lift"
of f (ie a lifting of f to R wrt the covering
map eit:R −> S^1.)

and the rotation number is the limit of

(*) (F^n(t) − t)/n,

(where F^n denotes iterated composition).

It follows that

eit(F(F(t)) = f(eit(F(t)) = f(f(eit(t));

that is, F^2 is a lift of f^2. If we're willing
t