

Re: circle homeomorphism

Source: <http://sci.tech--archive.net/Archive/sci.math/2005-10/msg02068.html>

- *From:* David C. Ullrich <ullrich@xxxxxxxxxxxxxxxxxxxx>
 - *Date:* Wed, 19 Oct 2005 06:13:08 -0500
-

On Tue, 18 Oct 2005 20:33:05 +0100, "Thomas Novascott"
<Thomas.novascott@xxxxxxxxxxxxxxxxxxxx> wrote:

>
>"David C. Ullrich" <ullrich@xxxxxxxxxxxxxxxxxxxx> wrote in message
>news:8ul711p6ljvhud9v55hubqsl8t7o3qq47f@xxxxxxxxxxx
>> On Sun, 16 Oct 2005 20:53:50 +0100, "Thomas Novascott"
>> <Thomas.novascott@xxxxxxxxxxxxxxxxxxxx> wrote:
>>
>>>
>>>"David C. Ullrich" <ullrich@xxxxxxxxxxxxxxxxxxxx> wrote in message
>>>news:ft6511tc7gd90vvjt159t7a12ebo5fi7ea@xxxxxxxxxxx
>>>> On Sun, 16 Oct 2005 11:46:21 +0100, "Thomas Novascott"
>>>> <Thomas.novascott@xxxxxxxxxxxxxxxxxxxx> wrote:
>>>>
>>>>> It seems we have an increasing homeomorphism F of \mathbb{R} ,
>>>>> with period 2π , such that
>>>>>
>>>>> $f(\text{eit}(t)) = \text{eit}(F(t))$
>>>>>
>>>>> (writing $\text{eit}(t) = \exp(2\pi i t)$); F is a "lift"
>>>>> of f (ie a lifting of f to \mathbb{R} wrt the covering
>>>>> map $\text{eit}:\mathbb{R} \rightarrow S^1$.)
>>>>>
>>>>> and the rotation number is the limit of
>>>>>
>>>>> $(*) (F^n(t) - t)/n$,
>>>>>
>>>>> (where F^n denotes iterated composition).
>>>>>
>>>>> It follows that
>>>>>
>>>>> $\text{eit}(F(F(t))) = f(\text{eit}(F(t))) = f(f(\text{eit}(t)))$;
>>>>>
>>>>> that is, F^2 is a lift of f^2 . If we're willing
>>>>> to believe that the limit $(*)$ exists the result
>>>>> you ask about follows easily...
>>>Thank you for your indepth explanation,
>>>i think i should have made it clearer, but i dont see how

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>>>eit(F(F(t)) = f(eit(F(t)) = f(f(eit(t)));
>>>
>>>proves $p(f^n) = n.p(f)$
>>>(sorry for not making this clearer)
>>
>> It proves it for $n = 2$. Since F^2 is a lift
>> of f^2 it follows that
>>
>> $p(f^2) = \lim ((F^2)^n(t) - t)/n$
>>
>> $= \lim (F^{2n}(t) - t)/n$
>>
>> $= 2 \lim (F^{2n}(t) - t)/(2n)$
>>
>> $= 2 \lim (F^n(t) - t)/n,$
>>
>> where the last equality is because if a sequence
>> converges then any subsequence converges to the
>> same limit.
>>
>seems i forgot the last statement! i see this proof would work for $n > 0$ and
> $n=0$ is obvious, out of curiosity could i use this approach to try and solve
> $n < 0$?

I've already posted a proof for $n = -1$ in this thread.

(That proof depended on the uniform convergence of that limit, which I had not proved at that point, but which I have proved since then. In this thread.)

David C. Ullrich

• **References:**

- ◆ **circle homeomorphism**
◇ From: Dumdum project
- ◆ **Re: circle homeomorphism**
◇ From: David C . Ullrich
- ◆ **Re: circle homeomorphism**
◇ From: David C . Ullrich
- ◆ **Re: circle homeomorphism**
◇ From: Thomas Novascott
- ◆ **Re: circle homeomorphism**
◇ From: David C . Ullrich
- ◆ **Re: circle homeomorphism**
◇ From: Thomas Novascott

Re: circle homeomorphism

- Prev by Date: *Re: Is there any body know the definition of normal tree in graph theory?*
- Next by Date: *Re: Sum of periodic functions with incommensurate periods*
- Previous by thread: *Re: circle homeomorphism*
- Next by thread: *Re: Fourier transform and the like*
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