

Re: prove it if you can

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Proginoskes wrote:

José Carlos Santos wrote:

[...]
Let s be the square root of 2. I don't know whether s^s is rational or not. If it is, then you're done. Otherwise, consider

$$(s^s)^s = s^{\{s^2\}} = s^2 = 2.$$

So, if s^s is irrational, you have the example that you want.

For the record, s^s is not only irrational, but transcendental. This is a consequence of Lindemann's Theorem.

How so? All I know about Lindemann's Theorem is what I googled for twenty seconds or so ago, and I can't see how it implies that. The theorem tells us that if a is algebraic, then $\exp(a)$ is transcendental. But s^s is $\exp(\ln(s)s)$, and surely $\ln(s)s$ isn't algebraic.