Re: Birthday problem for such non−uniform birthday probabilities

Source: http://sci.tech−archive.net/Archive/sci.math/2005−11/msg03110.html

- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Mon, 21 Nov 2005 12:49:25 −0500

On Mon, 21 Nov 2005 10:30:59 −0600, "Doug" <nospam@xxxxxxxxxx> wrote:

>
>"David M Einstein" <Deinst@xxxxxxxxx> wrote in message
>news:1132586540.046923.115340@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
>>
>> Doug wrote:
>>> "Helmut Zeisel" <helmut.zeisel@xxxxxx> wrote in message
>>> news:dls5fv.5lg.1@xxxxxxxxxxxxxxxxx
>>> > Consider the birthday problem:
>>> >
>>> > There are n randomly chosen person in a room. What is the proabability
>>> > that there exist k persons who have birthday on the same day.
>>> >
>>> > I know how to compute the probability assuming a uniform birthday
>>> > distribution.
>>> >
>>> > It is reasonable that this probability increases for a non−uniform
>>> > birthday distribution.
>>> >
>>> > Where can I find a proof for this result?
>>
>> Blom, D. (1973), "A birthday problem", American Mathematical Monthly,
>> vol. 80, pp. 1141−1142
>>
>>> >
>>> > Helmut
>>>
>>> You won't, as it is also reasonable that this probability decreases for a
>>> non−uniform
>>> distribution.
>>>
>>> You need to specify the "non−uniform" distributions to determine if it
>>> increases/decreases.
>>
>> I would be very interested in seeing a distribution that decreases the
>> probability of
>> a match.

>>
>
>simple− define "distribution" as sequential days beginning on Jan 1
>

Wrong.

Remember, no matter what distribution you specify, we are selecting randomly from it, so if k>1, repetitions are always possible.

If the distribution is not uniform, the probability of a match in k sample elements, where n>=k>1, is always greater than it would be if the distribution was uniform,

It's not hard to prove this in general, but as an example, assume a 2 day year and let n>=k=2.

Number the days of the year 1,2.

(Note that for this type of year, every day is either New Year's Day or New Year's Eve)

Let p1, p2 be the probabilities of having a birthday on days 1,2 respectively.

Since n>=k=2, the probability of a match can be calculated exactly as p1^2+p2^2.

Since p1, p2 are nonnegative,

p1^2+p2^2 >= 2*p1*p2 with equality iff p1=p2.

Then, adding p1^2+p2^2 to both sides and factoring, we get

2(p1^2+p2^2) >= (p1+p2)^2

Since p1+p2=1, we get

p1^2+p2^2 >= 1/2 with equality iff p1=p2.

Hence if the distribution is uniform, the probability of a match is exactly 1/2, otherwise it's strictly greater than 1/2.

quasi
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- ***Follow−Ups***:
  - ♦ ***Re: Birthday problem for such non−uniform birthday probabilities***
    - ◊ *From:* Doug

- ***References***:
  - ***Birthday problem for such non−uniform birthday probabilities***
    - *From:* Helmut Zeisel
  - ***Re: Birthday problem for such non−uniform birthday probabilities***
    - *From:* Doug
  - ***Re: Birthday problem for such non−uniform birthday probabilities***
    - *From:* David M Einstein
  - ***Re: Birthday problem for such non−uniform birthday probabilities***
    - *From:* Doug

- Prev by Date: ***Re: Well Ordering the Reals***
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