

Re: irreducible polynomials

Source: <http://sci.tech-archive.net/Archive/sci.math/2005-11/msg03910.html>

- *From:* Gerry Myerson <gerry@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>
 - *Date:* Fri, 25 Nov 2005 01:05:48 GMT
-

In article

<6753587.1132828228028.JavaMail.jakarta@xxxxxxxxxxxxxxxxxxxxxxxx>, eugene <jane1806@xxxxxxx> wrote:

- > I am stuck with the proof of the following fact:
- >
- > Prove that for every prime power q , one has that for all n there exists and
- > irreducible polynomial of degree n .
- >
- > At the beginning the proof suggest the following identity
- >
- > $\prod_{k=1}^{\infty} (1/(1-x^k)^{I(k)}) = \sum_{n=1}^{\infty} q^n x^n$
- >
- > where $I(k)$ —the number of irreducible polynomials of degree k (we are working
- > in our field F_q).
- > Could you please explain me this identity.
- > Thanks in advance

I hate to harp on typos, but the word you are looking for is "irreducible."

Anyway, the right side of the identity, the coefficient of x^n is the number of monic polynomials of degree n , right? Now, each such polynomial has a unique expression as a product of (monic) irreducible polynomials, right?

On the left side,

$$1 / (1 - x^k)^{I(k)} = (1 + x^k + x^{(2k)} + \dots)^{I(k)}$$

Now all you have to do is identify each term in the expansion of $\prod_{k=1}^{\infty} (1 + x^k + x^{(2k)} + \dots)^{I(k)}$ with a factorization into irreducibles. Can you see it?

—
Gerry Myerson (gerry@xxxxxxxxxxxxxxxxxxxx) (i -> u for email)

Re: irredicible polynomials

- **References:**

- ◆ ***irredicible polynomials***

- ◆ *From:* eugene

- Prev by Date: ***Re: Group actions***

- Next by Date: ***$AX+BY+CX+D=A'X'+B'Y'+C'X'+D'$***

- Previous by thread: ***irredicible polynomials***

- Next by thread: ***sparse matrix inversion algorithm***

- Index(es):

- ◆ ***Date***

- ◆ ***Thread***