Re: Defining "<" for the rationals

Source: http://sci.tech−archive.net/Archive/sci.math/2005−11/msg04156.html

- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Sat, 26 Nov 2005 17:16:19 −0500

On Sat, 26 Nov 2005 13:50:23 −0500, quasi <quasi@xxxxxxxx> wrote:

>On Sat, 26 Nov 2005 13:05:21 −0500, quasi <quasi@xxxxxxxx> wrote:
>
>>On Sat, 26 Nov 2005 17:33:25 +0100, Jannick Asmus
>><jannick.news@xxxxxx> wrote:
>>
>>>On 26.11.2005 17:21, Michael Stemper wrote:
>>>> I've been looking at the rational numbers as an equivalence relation.
>>>> I've been able to show that the definitions of multiplication and addition
>>>> give the same answer, regardless of which member of an equivalance set
>>>> is chosen. I wanted to also define the "<" relation, but hit a stumbling
>>>> block. Most of the time, you can say (a,b)<(c,d) iff ad<bc. However, this
>>>> falls apart if b or d is negative.
>>>>
>>>> I could say "chose a member of [(a,b)] with a positive second element,"
>>>> or I could say "if b<0, change the signs of a and b." Neither of these
>>>> seem particularly elegant. Is there a cleaner way to define the less
>>>> than relation on rationals?
>>>>
>>>
>>>If you require the relation < to be compatible with addition in the
>>>obvious way, you could reduce the situation by defining when a rational
>>>is positive.
>>>
>>>J.
>>
>>And once the concept of positive has been defined, then, assuming
>>subtraction has also been defined, you can define < by requiring
>>
>> (a,b) < (c,d) if (c,d) − (a,b) is positive.
>>
>>The steps:
>>
>>(1) Call (a,b) positive if ab>0
>>
>>(2) Define negation by: −(a,b)=(−a,b).
>>
>>(3) Define subtraction by: (c,d)−(a,b)=(c,d)+(−(a,b))

>>
>>(4) Define < by: (a,b) < (c,d) if (c,d) − (a,b) is positive.
>>
>>Of course, you have to show for each of the above that the resulting
>>class doesn't depend on the chosen representatives.
>>
>>quasi
>
>
>Of course, you could define < first by:
>
>(a,b)<(c,d) if (a*d−b*c)*(b*d)>0

correction:

(a,b)<(c,d) if (b*c−a*d)*(b*d)>0
>
>But such a definition is kind of artificial −− it hides what's really
>going on.
>
>quasi

quasi
.

- ***References***:
  - ♦ ***Defining "<" for the rationals***
    - ◊ *From:* Michael Stemper
  - ♦ ***Re: Defining "<" for the rationals***
    - ◊ *From:* Jannick Asmus
  - ♦ ***Re: Defining "<" for the rationals***
    - ◊ *From:* quasi
  - ♦ ***Re: Defining "<" for the rationals***
    - ◊ *From:* quasi

- Prev by Date: ***Re: Attiude with quaternions−***
- Next by Date: ***Re: Attiude with quaternions−−***
- Previous by thread: ***Re: Defining "<" for the rationals***
- Next by thread: ***Re: Defining "<" for the rationals***
- Index(es):
  - ♦ ***Date***
  - ♦ ***Thread***