

Re: Cardinality of the surreals

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- *From:* "Hero" <Hero.van.Jindelt@xxxxxx>
 - *Date:* 1 Dec 2005 14:05:57 -0800
-

Dik T. Winter wrote:

> In article <1133381969.897359.172680@xx> "Hero"
<Hero.van.Jindelt@xxxxxx> writes:

> ...

>> Christopher wrote to the question

>>> I'm not sure what surreals are,

>>

>>>> Conway's numbers from "On Numbers And Games". Basically, (L,R) is a

>>>> number iff L and R are numbers, and no element of R is less than or

>>>> equal to an element of L.

>

> That one is wrong. L and R are sets of numbers, not numbers (as the

> remainder: "no element" clarifies).

>

>> to what i refer to. $(3 | 4)$ is the surreal which given by the set of

>> all numbers smaller than 3 and the numbers 3 and 4 and all numbers

>> bigger than 4.

>

> This one is basically right. But you should remember that on the left

> and right of the vertical bar there is actually a set (in this case a

> singleton). So actually the "3" above might stand for the set of all

> numbers less than or equal to 3.

>

>> Now Peter wrote

>>> Definition: If L and R are two sets of surreal numbers and no member of

>>> R is less than or equal to any member of L then $\{ L | R \}$ is a surreal

>>> number.

>

> This is indeed the correct definition.

.....

> There is a single definition: given two sets of surreal numbers L and R

> such that every element of L is less than or equal to every element of R,

> the number $\{L|R\}$ is a new surreal number.

> However, to do that we need to define \leq .

> That is simple, given two surreals $x_1 = \{L_1|R_1\}$ and $x_2 = \{L_2|R_2\}$ we

> say that $x_1 \leq x_2$ if and only if there is (no x_2 in $R_2 \leq x_1$ and x_2

> \leq no x_1 in L_1).

>.....

Thanks Dik for Your patience with me and explaining according to my

Re: Cardinality of the surreals

problems, a book or a website never can do. Just to confirm, if i'm right now: The starting point is to define some surreals and as i want to talk about the ordinary reals as well i will differ them with an "s" or an "r".

>>From these surreal numbers like zero (s) or one (s), two (s) one advances to other surreal numbers like $(0(s) | 1(s))$ or $(1(s) | 1(s))$ of the form $(L | R)$, where R and L denote surreal numbers. The expression $(L | R)$ is defined as

a set of surreal numbers, actually a union of two sets of surreals which satisfy some conditions with respect to the ordering in these numbers. Now regarding the starting point, one can see, that the first numbers defined satisfy this definition too. Now a set of surreal numbers of this form and with given conditions form a surreal number as well. That is not different to Peano in this respect, the natural numbers are sets of numbers as well, for example Peano $2 = \{ \{ \} , \{ \{ \} \} \} = \{ \{ \} , 1 \} = \{ 0 , 1 \}$, all not surreal numbers.

Now, as You wrote to one of my sentences, that it is basically right, i feel encouraged to try a better formulation of what i'm looking for. Is this an isomorphism between specially constructed sets of real numbers and surreal numbers:

$|\mathbb{R} \longrightarrow 0(s)$

Now basically rays of the real number line or given by

$\{ r, r \text{ element } |\mathbb{R} \text{ and } r \text{ smaller or equal } a, a \text{ element } |\mathbb{R} \} \longrightarrow (A |)$

$\{ r, r \text{ element } |\mathbb{R} \text{ and } r \text{ bigger or equal } b, b \text{ element } |\mathbb{R} \} \longrightarrow (| B)$

Now $|\mathbb{R}$ with an open intervall cut out

$\{ r, r \text{ element } |\mathbb{R} \text{ and } r \text{ smaller or equal } d, d \text{ element } |\mathbb{R} \}$ union with

$\{ r, r \text{ element } |\mathbb{R} \text{ and } r \text{ bigger or equal } g, g \text{ element } |\mathbb{R} \} \longrightarrow (D | G)$

or does this goes wrong somewhere when coming to arithmetic ?

Regards

Hero

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• Follow-Ups:

◆ Re: Cardinality of the surreals

◇ From: Peter Webb

• References:

◆ Re: Cardinality of the surreals

◇ From: Hero

◆ Re: Cardinality of the surreals

◇ From: Nathan

◆ Re: Cardinality of the surreals

◇ From: Hero

Re: Cardinality of the surreals

◆ *Re: Cardinality of the surreals*

◇ *From:* Dik T. Winter

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